BASIC CONCEPTS/DEFINITIONS OF FLUID MECHANICS
(by Marios M. Fyrillas)

1. **Density (πυκνότητα)**

   Symbol: ρ

   Units of measure: $\mathrm{kg/m^3}$

   Equation: $\rho = \frac{m}{\cancel{V}}$ $(m \equiv \text{ mass}, \ \cancel{V} \equiv \text{ volume})$

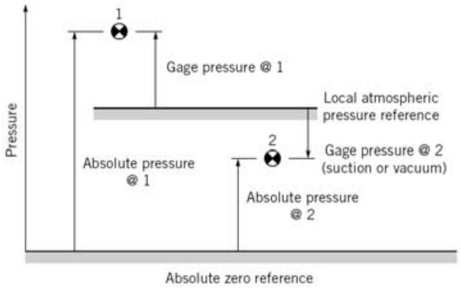
2. **Pressure (πίεση)**

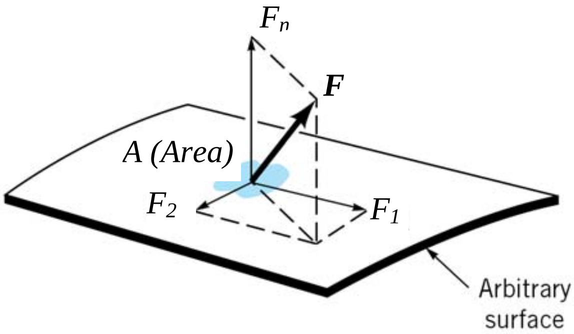
   Alternative definition: Normal Stress (see right figure)

   Symbol: p

   Units of measure: $\mathrm{N/m^2}$

   Measurement of pressure: see left figure





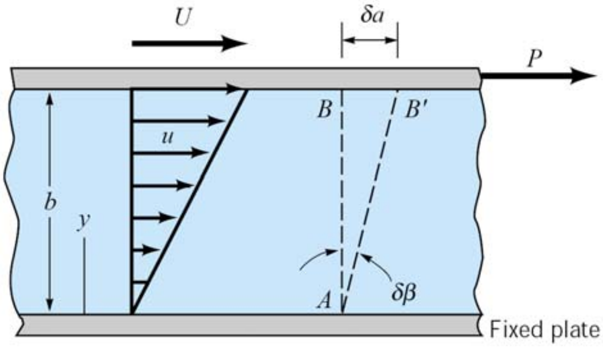
   Equation: $p = \frac{F_n}{A}$ $(F_n \equiv \text{ normal force}, \ A \equiv \text{ area})$

3. **Shear Stress (Διατμητική τάση)**

   Symbol: τ (see right figure above)

   Units of measure: $\mathrm{N/m^2}$

   Equation: $\tau = \frac{F_{1or2}}{A}$ $(F_{1or2} \equiv \text{ tangential force}, \ A \equiv \text{ area})$



   For a Newtonian fluid the relation between the shear stress and the viscosity is as follows:

$$\tau = \mu \frac{du}{dy}$$

   For a linear velocity profile (see figure above):

$$\tau = \mu \frac{du}{dy} = \mu \frac{\delta u}{\delta y} = \mu \frac{U-0}{b-0} = \mu \frac{U}{b}$$

4. **Viscosity (Ιξώδες)**
   Symbol: μ
   Units of measure: $\mathrm{N \cdot s / m^2 = kg/(m \cdot s)}$
   Equation: There is no fundamental definition

5. **Pressure head**
   Symbol: h or H
   Units of measure: m
   Equation: $h = \dfrac{p}{\rho g}$ $\quad (p \equiv \text{ pressure, } \rho \equiv \text{ density, } g = \text{acceleration of gravity } 9.81\ \mathrm{m/s^2})$

6. **Mass Flow Rate**
   Symbol: $\dot{m}$
   Units of measure: kg/s
   Equation: For uniform flow

$$\dot{m} = \rho u A \text{ where } \begin{cases} \rho \equiv \text{density, } A \equiv \text{area,} \\ u \equiv \textbf{component of velocity normal to the cross-sectional area} \end{cases}$$

7. **Volumetric Flow Rate**
   Symbol: $\dot{V}$ or Q
   Units of measure: m^3/s
   Equation: For uniform flow $Q = uA = \dfrac{\dot{m}}{\rho}$ $\quad (\rho \equiv \text{density, } u \equiv \text{normal velocity, } A \equiv \text{area})$

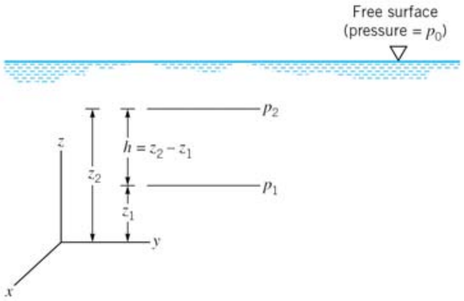
8. **Reynolds number**
   Symbol: $\mathrm{Re} \equiv \dfrac{\text{inertial forces}}{\text{viscous forces}}$
   Units of measure: Dimensionless. It represents the ratio of inertial to viscous forces

   Equation: $\mathrm{Re} = \dfrac{\rho u l}{\mu}$ $\quad \begin{cases} \rho \equiv \text{density, } u \equiv \text{velocity,} \\ \mu \equiv \text{viscocity, } l \equiv \text{characteristic length} \end{cases}$

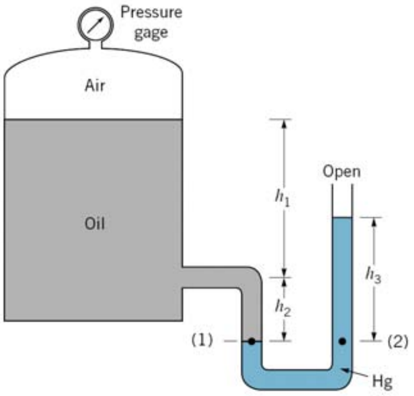
9. **Froyde number**
   Symbol: $\mathrm{Fr} \equiv \dfrac{\text{inertial forces}}{\text{gravitational forces}}$
   Units of measure: Dimensionless. It represents the ratio of inertial to gravitational forces
   Equation: $\mathrm{Fr} = \dfrac{u}{\sqrt{g\,y}}$ $\quad \begin{cases} g \equiv \text{acceleration of gravity, } u \equiv \text{velocity,} \\ y \equiv \text{depth, Note: } \sqrt{g\,y} \equiv \text{ wave speed in shallow water} \end{cases}$

BASIC EQUATIONS

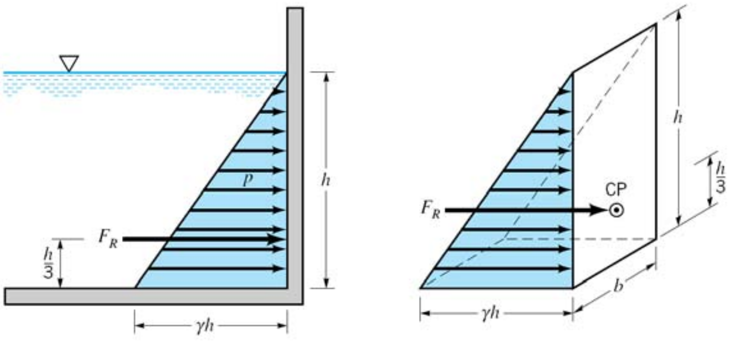
1. Fluid Statics



Basic Equation: $p_1 = \rho g h + p_2$ (see figure above)
For fluids at rest the pressure for two points that lie along the same vertical direction is the same, i.e. in configuration below $p_1 = p_2$



i. Hydrostatic Forces on Surfaces



(a) (b)

The magnitude of the resultant fluid force is equal to the volume of the pressure prism. It passes through the centroid CP (for vertical rectangular area is at $h/3$ (see right fig.)

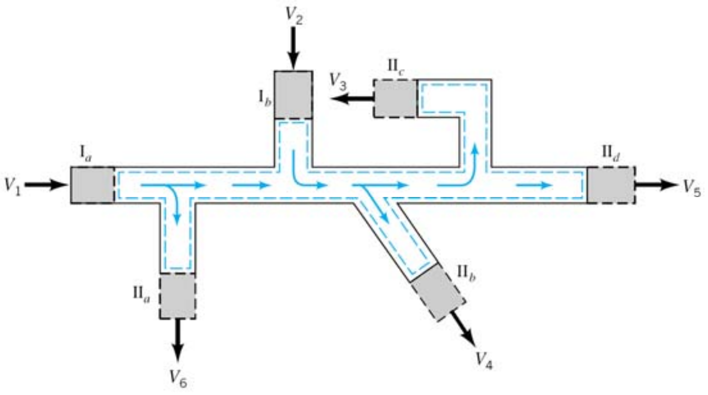
ii. Buoyancy (Άνωση)

Archimedes' principle states that the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward:

$$F_B = \rho g \mathcal{V} \quad (\mathcal{V} \equiv \text{volume of displaced fluid})$$

2. Fluid Dynamics

A basic concept is that of a control volume which is a volume in space



A typical control volume with more than one inlet and outlet

i. Conservation of Mass – The Continuity Equation

$$\frac{\mathrm{d}m_{CV}}{\mathrm{dt}} = \dot{m}_{in} - \dot{m}_{out}$$

For steady flow: $\sum \dot{m}_{out} = \sum \dot{m}_{in}$

ii. Conservation of Momentum – The Linear Momentum Equation

For steady flow: $\sum \mathbf{F} = \underbrace{\sum \dot{\mathbf{M}}_{out}}_{\text{mometum flux out}} - \underbrace{\sum \dot{\mathbf{M}}_{in}}_{\text{mometum flux in}}$

For one dimensional (uniform) flow becomes: $\sum \mathbf{F} = \sum \dot{m}_{out}\mathbf{u}_{out} - \sum \dot{m}_{in}\mathbf{u}_{in}$

Note: The momentum equation is a vector equation, i.e. it has three components.

Example: The x-momentum for a system with one inlet and one outlet it becomes

$$F_x = \dot{m}_{out}u_{out_x} - \dot{m}_{in}u_{in_x}$$

iii. Moment-of-Momentum Equation

For one-dimensional flow through a rotating machine, we obtain:

$$T_{shaft} = (-\dot{m}_{in})(\pm r_{in}V_{\theta in}) + \dot{m}_{out}(\pm r_{out}V_{\theta out})$$

This is similar to the momentum equation with the exception that the arm of the force is included. The sign is determined as follows: If the blade speed $U = r\omega$ is in the same direction with V_θ then, rV_θ is positive.

The shaft power, is related to torque by

$$\dot{W}_{shaft} = T_{shaft}\omega \,.$$

Hence, combining with the torque equation we obtain:

$$\dot{W}_{shaft} = (-\dot{m}_{in})(\pm r_{in}\omega\, V_{\theta in}) + \dot{m}_{out}(\pm r_{out}\omega\, V_{\theta out})$$

or

$$\dot{W}_{shaft} = (-\dot{m}_{in})(\pm U_{in}V_{\theta in}) + \dot{m}_{out}(\pm U_{out}V_{\theta out})$$

iv. Conservation of Mechanical Energy - Bernoulli Equation

$$p + \frac{1}{2}\rho V^2 + \rho g z = \text{ constant along a streamline}$$

Assumptions:
a. viscous effects (fluid friction) are negligible
b. flow is incompressible
c. steady flow
d. valid along a streamline (tangent to the velocity field)
e. $V^2 \equiv$ square of the magnitude of the velocity

Another form that is commonly used is in terms of head:

$$\underbrace{\frac{p}{\rho g}}_{\substack{\text{pressure}\\ \text{head (h)}}} + \underbrace{\frac{V^2}{2g}}_{\substack{\text{velocity}\\ \text{head}}} + z = \text{ constant along a streamline} = H \text{ (total head)}$$

3. Fluid dynamics and thermodynamics

$$\dot{m}\left[\breve{u}_{out} - \breve{u}_{in} + \left(\frac{p}{\rho}\right)_{out} - \left(\frac{p}{\rho}\right)_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}$$

This is the one-dimensional energy equation for steady-in-the-mean flow.
The energy loss due to friction losses is translated into a temperature rise or a heat loss. Hence the terms $\breve{u}_{out} - \breve{u}_{in} - \frac{\dot{Q}_{\substack{\text{net}\\ \text{in}}}}{\dot{m}}$ represent the losses. The energy equation takes the form

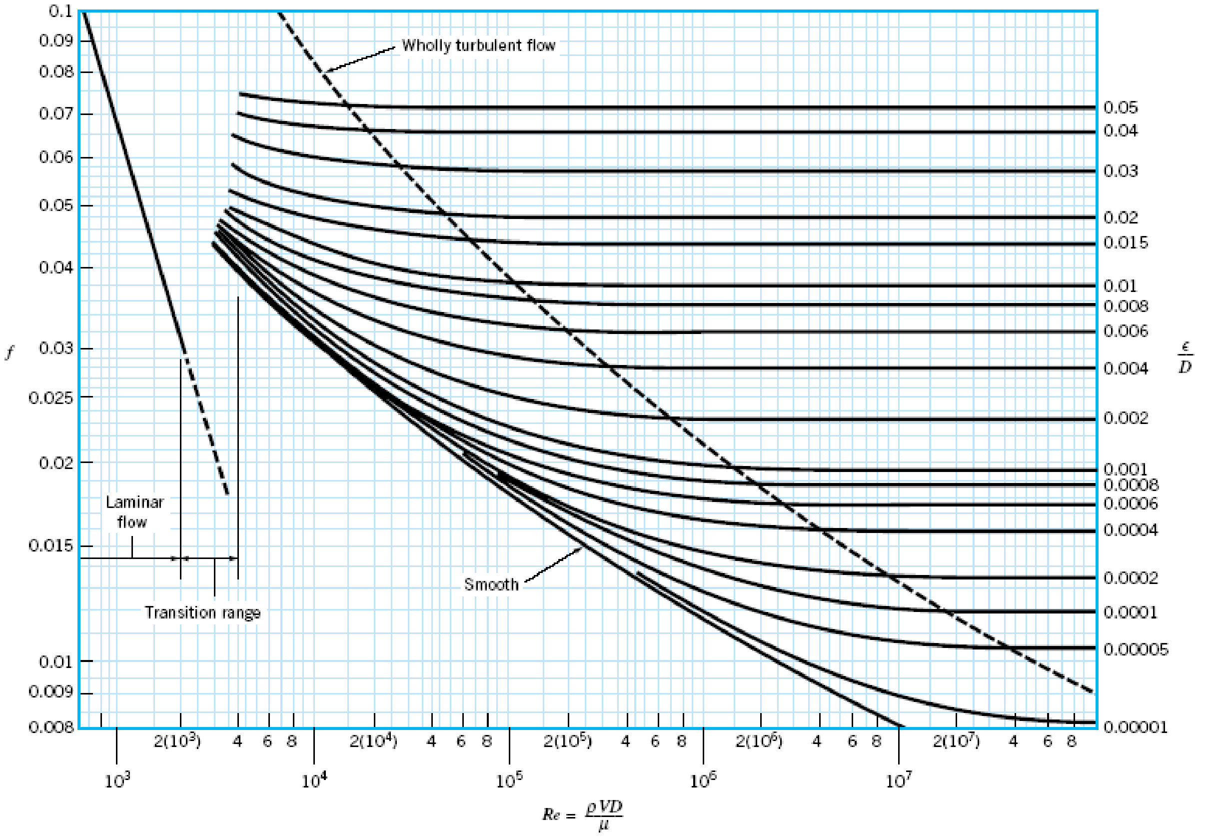
$$\frac{p_{out}}{\rho g} + \frac{V_{out}^2}{2g} + z_{out} + h_L = \frac{p_{in}}{\rho g} + \frac{V_{in}^2}{2g} + z_{in} + \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net out}}}}{\dot{m} g} \,,$$

where h_L is the head loss and can be obtained through the friction factor f:

$$h_L = \underbrace{f\,\frac{\ell}{D}\frac{V^2}{2g}}_{\text{major losses}} + \underbrace{K_L\frac{V^2}{2g}}_{\text{minor losses}} \,.$$

The head loss h_L has two contributions: head losses in the straight sections identified as major losses and the head loss in the different components, i.e. contractions, expansions, conical diffusers, bends, tees and unions. The friction factor f can be obtained from the Moody chart and, the loss coefficient K_L from figures/tables (see next pages).

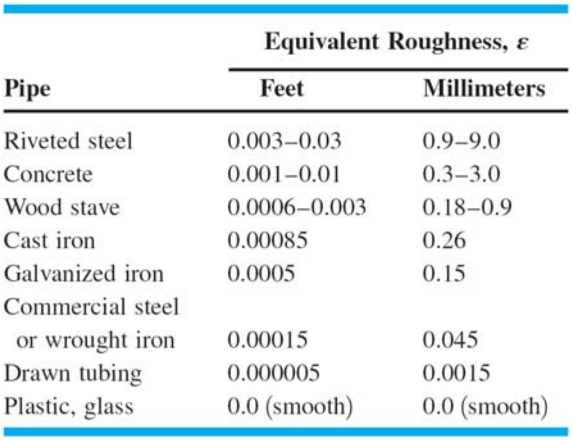
Calculation of Major Losses:



■ **FIGURE 8.20** **Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)**

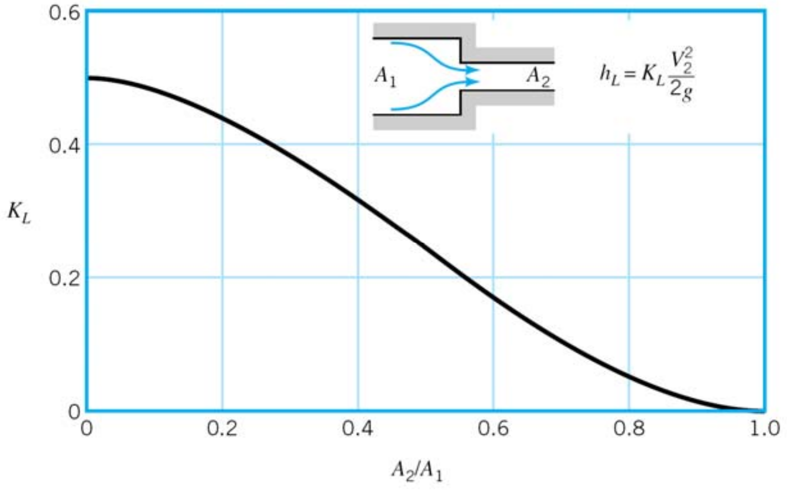
Figure: The Moody Chart

Table: Equivalent Roughness

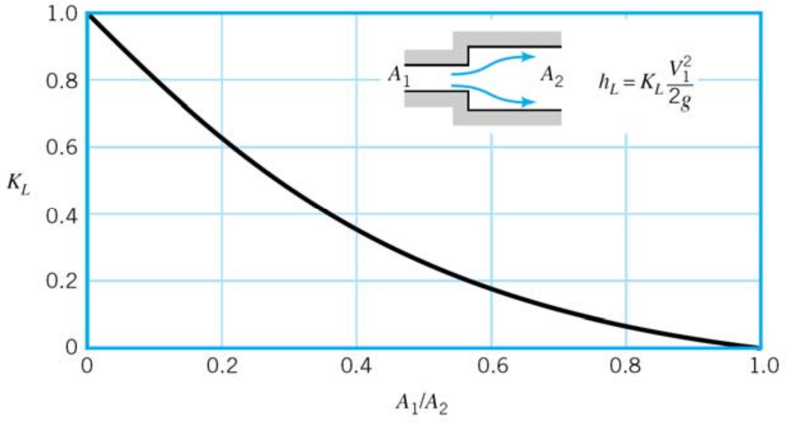


	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

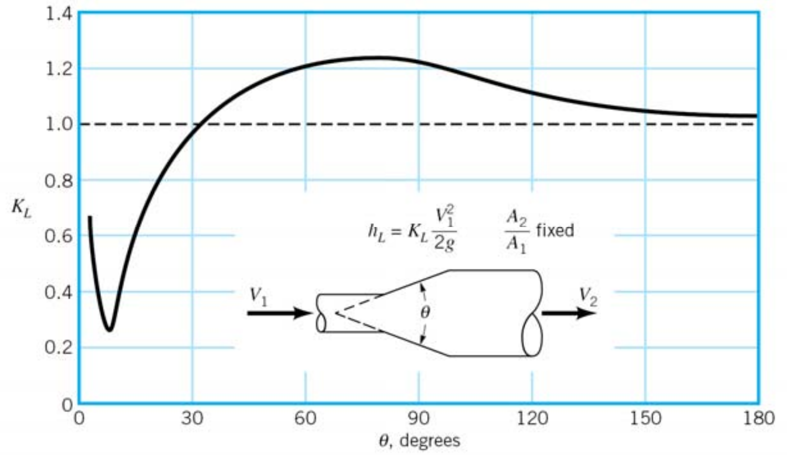
Calculation of Minor Losses:



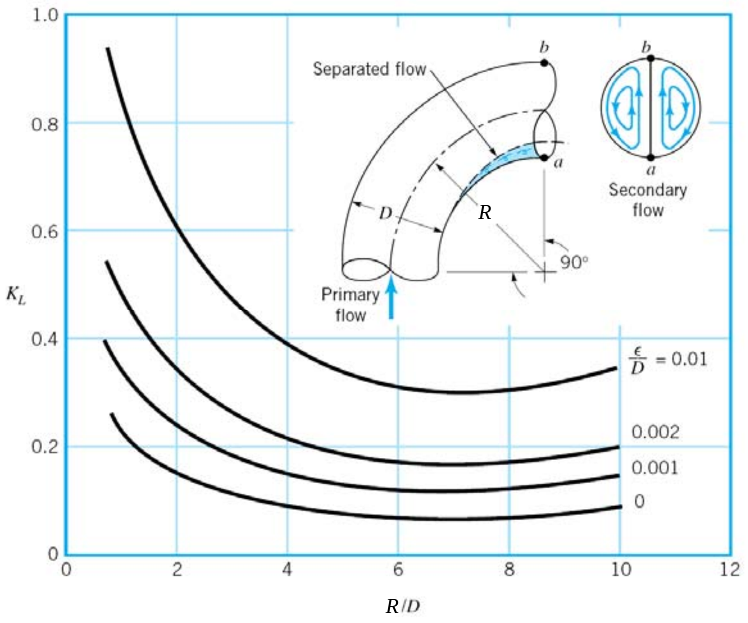
Sudden Contraction



Sudden Expansion

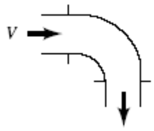
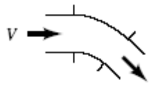
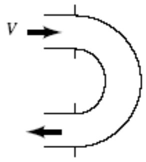
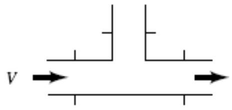
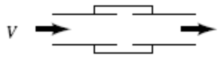


Conical Diffuser



90 bend with different radii

Table 1: **Loss Coefficients for Pipe Components** $\left(h_L = K_L \frac{V^2}{2g}\right)$

Component	K_L	
a. Elbows		
Regular 90°, flanged	0.3	
Regular 90°, threaded	1.5	
Long radius 90°, flanged	0.2	
Long radius 90°, threaded	0.7	
Long radius 45°, flanged	0.2	
Regular 45°, threaded	0.4	
b. 180° return bends		
180° return bend, flanged	0.2	
180° return bend, threaded	1.5	
c. Tees		
Line flow, flanged	0.2	
Line flow, threaded	0.9	
Branch flow, flanged	1.0	
Branch flow, threaded	2.0	
d. Union, threaded	0.08	
*e. **Valves**		
Globe, fully open	10	
Angle, fully open	2	
Gate, fully open	0.15	
Gate, $\frac{1}{4}$ closed	0.26	
Gate, $\frac{1}{2}$ closed	2.1	
Gate, $\frac{3}{4}$ closed	17	
Swing check, forward flow	2	
Swing check, backward flow	∞	
Ball valve, fully open	0.05	
Ball valve, $\frac{1}{3}$ closed	5.5	
Ball valve, $\frac{2}{3}$ closed	210	
f. Square edged exit	**1.0**	
Squared edged entrance	**0.5**	