11. IMPACT OF A JET

Introduction

Water turbines are widely used throughout the world to generate power. In the type of water turbine referred to as a Pelton[†] wheel, one or more water jets are directed tangentially on to vanes or buckets that are fastened to the rim of the turbine disc. The impact of the water on the vanes generates a torque on the wheel, causing it to rotate and to develop power. Although the concept is essentially simple, such turbines can generate considerable output at high efficiency. Powers in excess of 100 MW, and hydraulic efficiencies greater than 95%, are not uncommon. It may be noted that the Pelton wheel is best suited to conditions where the available head of water is great, and the flow rate is comparatively small. For example, with a head of 100 m and a flow rate of 1 m^3/s, a Pelton wheel running at some 250 rev/min could be used to develop about 900 kW. The same water power would be available if the head were only 10 m and the flow were $10 m^3/s$, but a different type of turbine would then be needed.

To predict the output of a Pelton wheel, and to determine its optimum rotational speed, we need to understand how the deflection of the jet generates a force on the buckets, and how the force is related to the rate of momentum flow in the jet. In this experiment, we measure the force generated by a jet of water striking a flat plate or a hemispherical cup, and compare the results with the computed momentum flow rate in the jet.

Description of Apparatus

Fig 11.1 shows the arrangement, in which water supplied from the Hydraulic Bench is fed to a vertical pipe terminating in a tapered nozzle. This produces a jet of water which impinges on a vane, in the form of a flat plate or a hemispherical cup.

The nozzle and vane are contained within a transparent cylinder, and at the base of the cylinder there is an outlet from which the flow is directed to the measuring tank of the

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[†]L A Pelton was an American engineer who, in the late 19th century, made extensive experiments using various shapes of buckets, with the aim of obtaining high efficiency. He devised a bucket shape which has a central splitter to divide the jet. His improved wheel was patented in 1880.

bench. As indicated in Fig 11.1, the vane is supported by a lever which carries a jockey weight, and which is restrained by a light spring. The lever may be set to a balanced position (as indicated by a tally supported from it) by placing the jockey weight at its zero position, and then adjusting the knurled nut above the spring. Any force generated by impact of the jet on the vane may now be measured by moving the jockey weight along the lever until the tally shows that it has been restored to its original balanced position.

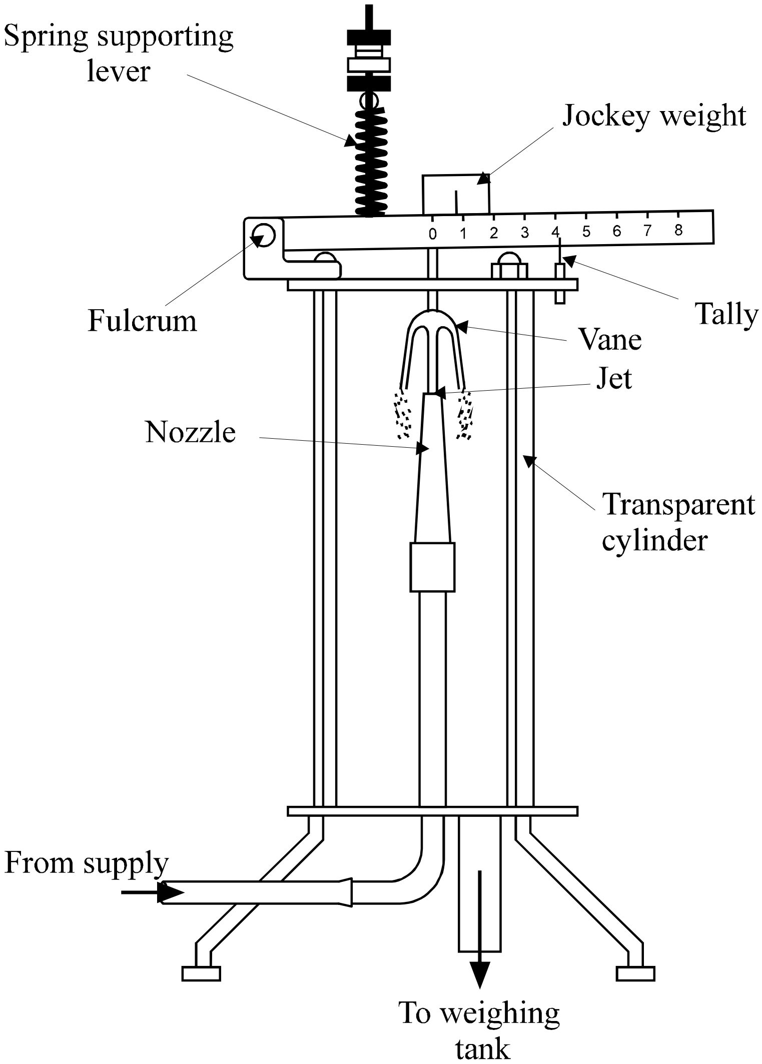

Fig 11.1 Arrangement of Apparatus

Theory of the Experiment

The equation of momentum is discussed in Section 1.3 of Chapter 1. Consider how it applies to the case shown schematically in Fig 11.2, which shows a jet of fluid impinging on a symmetrical vane.

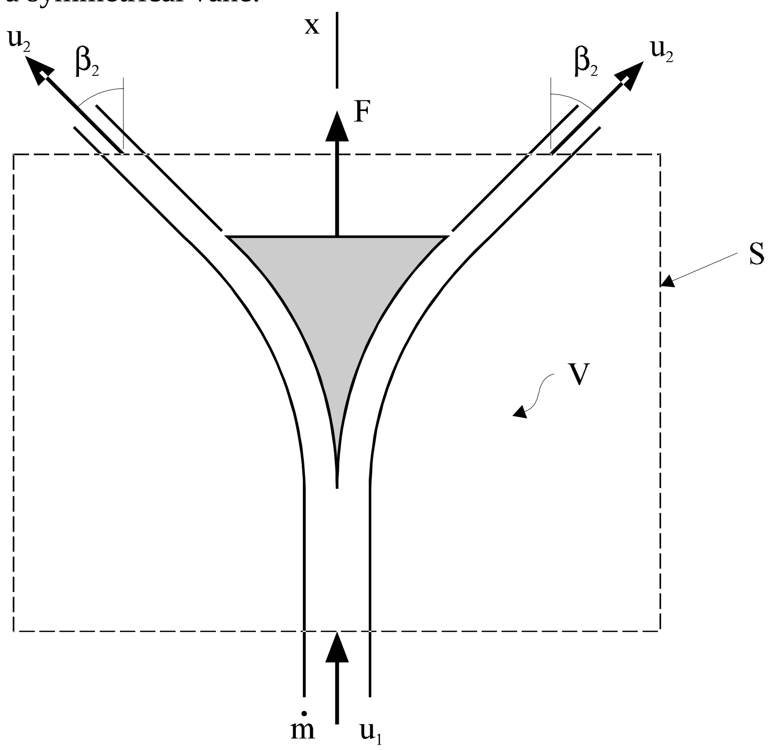

Fig 11.2 Sketch of jet impinging on a vane

Let the mass flow rate in the jet be $\dot{m}$. Imagine a control volume V, bounded by a control surface S which encloses the vane as shown. The velocity with which the jet enters the control volume is u_1, in the x-direction. The jet is deflected by its impingement on the vane, so that it leaves the control volume with velocity u_2, inclined at an angle β_2 to the x-direction. Now the pressure over the whole surface of the jet, apart from that part where it flows over the surface of the vane, is atmospheric. Therefore, neglecting the effect of gravity, the changed direction of the jet is due solely the force generated by pressure and shear stress at the vane's surface. If this force **on the jet** in the direction of x be denoted by F_j, then the momentum equation in the x-direction is

$$F_j = \dot{m}\left(u_2 \cos\beta_2 - u_1\right) \tag{11.1}$$

The force F **on the vane** is equal and opposite to this, namely

$$F = \dot{m}\left(u_1 - u_2 \cos\beta_2\right)$$

(11.2)

For the case of a flat plate, $\beta_2 = 90°$, so that $\cos \beta_2 = 0$. It follows that

$$F = \dot{m} u_1$$

(11.3)

is the force on the flat plate, irrespective of the value of u_2.

For the case of a hemispherical cup, we assume that $\beta_2 = 180°$, so that $\cos\beta_2 = -1$, and

$$F = \dot{m}(u_1 + u_2)$$

(11.4)

If we neglect the effect of change of elevation on jet speed, and the loss of speed due to friction over the surface of the vane, then $u_1 = u_2$, so

$$F = 2\dot{m} u_1$$

(11.5)

is the maximum possible value of force on the hemispherical cup. This is just twice the force on the flat plate.

Returning now to Fig 11.2, the rate at which momentum is entering the control volume is $\dot{m} u_1$. We may think of this as a rate of flow of momentum in the jet, and denote this by the symbol J, where

$$J = \dot{m} u_1$$

(11.6)

For the flat plate, therefore, we see from Equation (11.3) that

$$F = J$$

(11.7)

and for the hemispherical cup the maximum possible value of force is, from Equation (11.5)

$$F = 2J \qquad \textbf{(11.8)}$$

In the SI system the units of $\dot{m}$ and u are

$$\dot{m}\ [kg/s] \quad \text{and} \quad u\ [m/s]$$

In an equation such as (11.3), then, the units of force F are

$$F\ [kg/s].[m/s] \ \text{ or } \ [kg\ m/s^2] \ \text{ or } \ [N]$$

Experimental Procedure

The apparatus is first levelled and the lever brought to the balanced position (as indicated by the tally), with the jockey weight at its zero setting. Note the weight of the jockey, and the following dimensions: diameter of the nozzle, height of the vane above the tip of the nozzle when the lever is balanced, and distance from the pivot of the lever to the centre of the vane.

Water is then admitted through the bench supply valve, and the flow rate increased to the maximum. The force on the vane displaces the lever, which is then restored to its balanced position by sliding the jockey weight along the lever. The mass flow rate is established by collection of water over a timed interval. Further observations are then made at a number reducing flow rates. About eight readings should suffice.

The best way to set the conditions for reduced flow rate is to place the jockey weight exactly at the desired position, and then to adjust the flow control valve to bring the lever to the balanced position. The condition of balance is thereby found without touching the lever, which is much easier than finding the point of balance by sliding the jockey weight. Moreover, the range of settings of the jockey position may be divided neatly into equal steps.

The experiment should be run twice, first with the flat plate and then with the hemispherical cup.

Results and Calculations

Diameter of nozzle, D	= 10.0 mm
Cross sectional area of nozzle, $A = \pi D^2/4$	$= 78.5\ mm^2 = 7.85 \times 10^{-5}\ m^2$
Height of vane above nozzle tip s	= 35 mm = 0.035 m
Distance from centre of vane to pivot of lever L	= 150 mm
Mass of jockey weight, M	= 0.600 kg
Weight of jockey weight, W = Mg	$= 0.600 \times 9.81 = 5.89$ N

When the jockey weight is moved a distance y mm from its zero position, the force F on the vane which is required to restore balance is given by:

$$F \times 150 = W \times y$$

Inserting the value of W, namely 5.89 N, gives

$$F = \frac{5.89 \times y}{150} \quad \text{or} \quad F = 0.03924\, y \text{ N}$$

The mass flow rate $\dot{m}$ in the jet is found by timing the collection of a known mass of water. The velocity u_1 of the jet as it leaves the nozzle is found from the volumetric flow rate Q and the cross sectional area A of the nozzle. The velocity u_0 with which the jet strikes the vane is slightly less than u_1 because of the deceleration due to gravity. This effect may be calculated from the expression

$$u_0^2 = u_1^2 - 2gs$$

Inserting the value s = 0.035 m leads to the result

$$u_0 = \sqrt{u_1^2 - 0.687} \text{ m/s}$$

Recorded values of quantity collected, measured time, and jockey displacement y are presented in Tables 11.1 and 11.2, together with the ensuing calculations. In the first line of Table 11.1, for example,

Mass flow rate $\dot{m}$ $$\frac{30}{62.2} = 0.482 \text{ kg/s}$$

Volumetric flow rate Q $\quad 0.482 \times 10^{-3} \ \text{m}^3/\text{s}$

Velocity at nozzle exit $u_1 = \dfrac{Q}{A} \quad \dfrac{0.482 \times 10^{-3}}{7.85 \times 10^{-5}} = 6.14 \text{ m s}$

Velocity at impact with vane $u_0 \quad \sqrt{u_1^2 - 0.687} = 6.08 \text{ m s}$

Momentum flow in jet at impact $J = \dot{m} u_0 \quad 0.482 \times 6.08 = 2.93 \text{ N}$

Force on vane F $\quad 0.03924 \times y = 0.3924 \times 74 = 2.90 \text{ N}$

Qty (kg)	**t (s)**	**y (mm)**	**$\dot{m}$ (kg/s)**	**u_1 (m/s)**	**u_0 (m/s)**	**J (N)**	**F (N)**
30	62.2	74	0.482	6.14	6.08	2.93	2.90
30	69.6	60	0.431	5.49	5.42	2.34	2.35
30	76.1	50	0.394	5.02	4.95	1.95	1.96
15	41.1	40	0.365	4.65	4.57	1.67	1.57
15	46.8	30	0.321	4.08	4.00	1.28	1.18
15	57.1	20	0.263	3.34	3.24	0.85	0.78
15	79.5	10	0.189	2.40	2.25	0.43	0.39

Table 11.1 Results for flat plate

Qty (kg)	**t (s)**	**y (mm)**	**$\dot{m}$ (kg/s)**	**u_1 (m/s)**	**u_0 (m/s)**	**J (N)**	**F (N)**
30	64.0	136	0.469	5.97	5.91	2.77	5.34
30	67.6	120	0.444	5.65	5.59	2.48	4.71
30	74.4	100	0.403	5.13	5.07	2.04	3.92
15	41.3	80	0.363	4.62	4.55	1.65	3.14
15	47.2	60	0.318	4.05	3.96	1.26	2.35
15	57.6	40	0.260	3.32	3.21	0.86	1.57
15	80.2	20	0.187	2.38	2.23	0.42	0.78

Table 11.2 Results for hemispherical cup

The results are shown in graphical form on Fig 11.3.

Discussion of Results

It is clear from Fig 11.3 that the force produced on each of the vanes is proportional to the momentum flow in the jet as it strikes the vane.

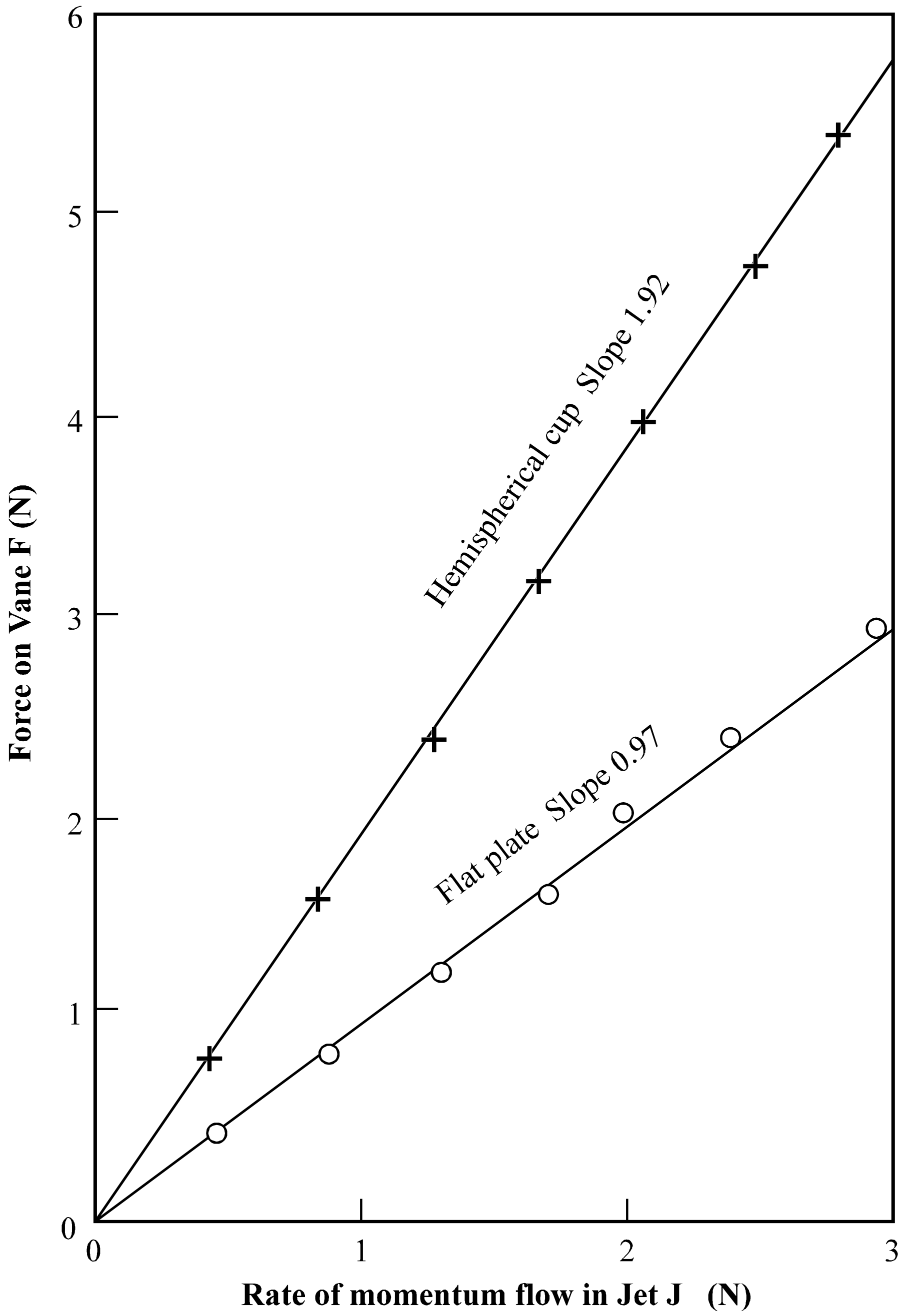

Fig 11.3 Force produced by a water jet striking a flat plate and a hemispherical cup

For the flat plate, the slope of the graph for is 0.97, as compared with the ideal value of 1.00. The discrepancy is possibly due to systematic experimental errors, such as a small error in the measured value of nozzle diameter. A further possibility lies in the behaviour of the jet after striking the vane. It forms a radial sheet which impinges on the inner wall of the surrounding cylinder, and then divides, some of the water flowing down the cylinder wall and the rest flowing upwards. Although visibility is impaired by the spray which is generated, it does seem that some water falls on to the top side of the vane. This would have the effect of producing a small momentum force in the downwards direction, so reducing the net upwards force on the vane.

For the hemispherical cup, the slope of the graph is 1.92, so that

$$F = 1.92J$$

The maximum possible force is shown by Equation (11.8) to be 2J, which occurs when the jet is deflected through 180° without energy loss. The ratio between measured force and the theoretical maximum may be regarded as an efficiency of the vane, 0.96 in this case, and will always be less that 1.00 because losses will always occur in practice.

Questions for Further Discussion

1. What suggestions have you for improving the apparatus?

2. What would be the effect on the calculated value of the vane efficiency of the following systematic errors of measurement:

   a) Mass of jockey weight in error by 0.001 kg.
   b) Distance L from centre of vane to pivot of lever in error by 1 mm.
   c) Diameter of water jet emerging from nozzle in error by 0.1 mm.

3. It has been assumed that the velocity in the jet is uniform over its cross section. How would the momentum flow in the jet be affected if this were not so? Consider, for example, a jet of cross sectional area A in which the velocity is $0.5u_1$ over half the area and $1.5u_1$ over the other half. The mass flow rate is the same as if the velocity were u_1 over the whole, namely

$$\dot{m} = \rho u_1 A$$

Show that the rate of momentum flow in the jet is

$$J = 1.25 \, \dot{m} u_1$$

i.e. 25% greater that if the velocity were uniform over the jet.

4. What would be the effect on the calculated force on the flat plate if the jet were to leave the plate not absolutely horizontal, but inclined upwards at an angle of 1°?

5. If the experiment were to be repeated with the vane in the form of a cone with an included angle of 60° (half angle 30°), how would you expect the results to appear on Fig 11.3?