AMEE 202
Introduction to Fluid Mechanics

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Homework Assignment on
Fluid Statics

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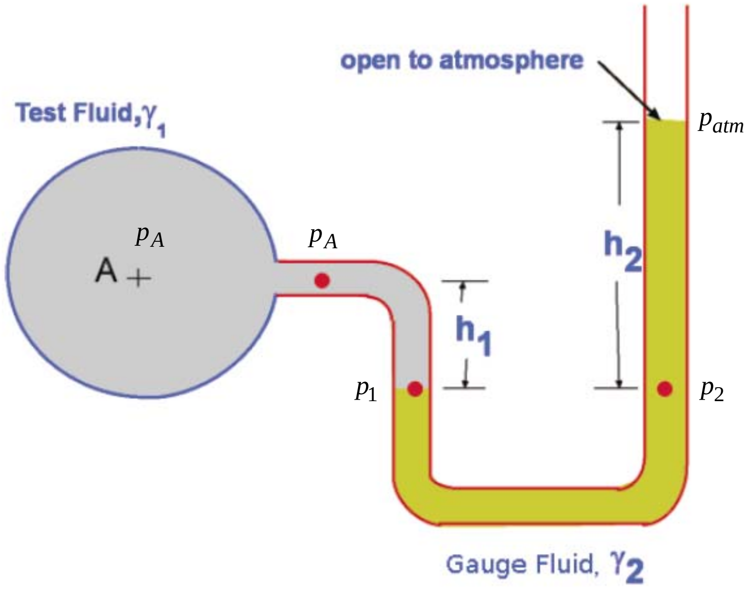
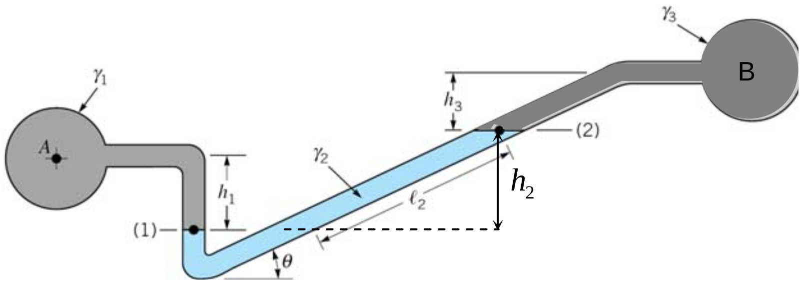
1. **Determine the relationship between pressure at point A and pressure at point B**



$$\left.\begin{array}{l} p_1 = p_A + \gamma_1 h_1 \\ \left.\begin{array}{l} p_1 = p_2 \\ p_2 = p_{atm} + \gamma_2 h_2 \end{array}\right\} \Rightarrow p_1 = p_{atm} + \gamma_2 h_2 \end{array}\right\} \Rightarrow p_{atm} + \gamma_2 h_2 = p_A + \gamma_1 h_1$$

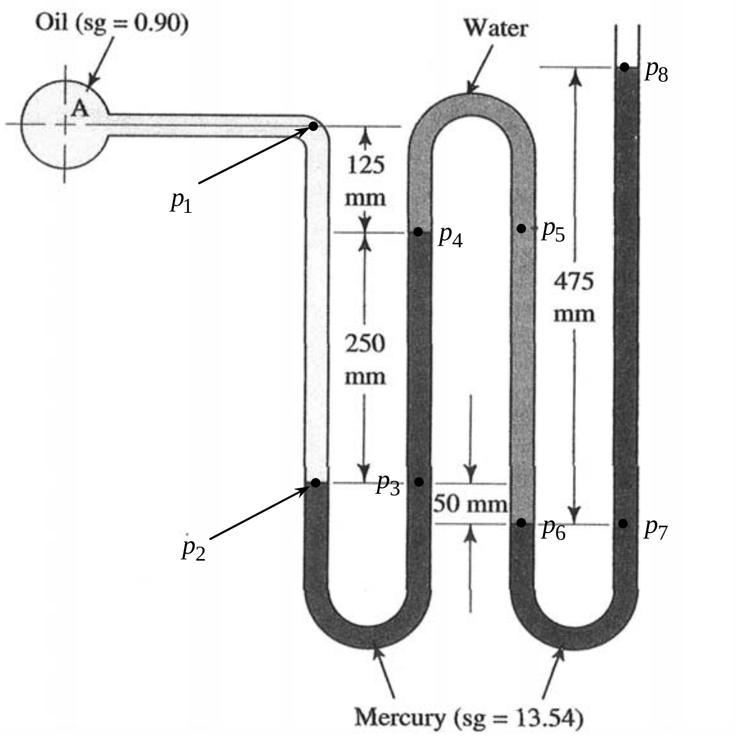
2. **The figure shows an inclined manometer, in which the distance ℓ_2 indicates the movement of the gauge fluid level. Determine an expression for the pressure difference between p_A and p_B.**



$$\left.\begin{aligned} p_2 &= p_B + \rho_3 g h_3 = p_B + \gamma_3 h_3 \\ p_1 &= p_2 + \rho_2 g h_2 = p_2 + \gamma_2 h_2 = p_B + \gamma_3 h_3 + \gamma_2 h_2 \\ p_1 &= p_A + \rho_1 g h_1 = p_A + \gamma_1 h_1 \end{aligned}\right\} \Rightarrow$$

$$p_B + \gamma_3 h_3 + \gamma_2 h_2 = p_A + \gamma_1 h_1$$
$$\Rightarrow p_A - p_B = \gamma_3 h_3 + \gamma_2 h_2 - \gamma_1 h_1 = \gamma_3 h_3 + \gamma_2 \ell_2 \sin\theta - \gamma_1 h_1$$

3. For the compound differential manometer in the figure calculate the pressure at the point A.



$$\left.\begin{array}{l} p_1 = p_A \\ p_2 = p_1 + \rho_{oil}\ g\ (0.125+0.25) = p_A + 900\times 9.81\times 0.375 = p_A + 3311 \\ \\ \left.\begin{array}{l} p_2 = p_3 \\ p_4 = p_3 - \rho_m g\times 0.25 \end{array}\right\} \Rightarrow p_4 = p_2 - 13540\times 9.81\times 0.25 = p_2 - 33207 \end{array}\right\} \Rightarrow p_4 = p_A - 29896$$

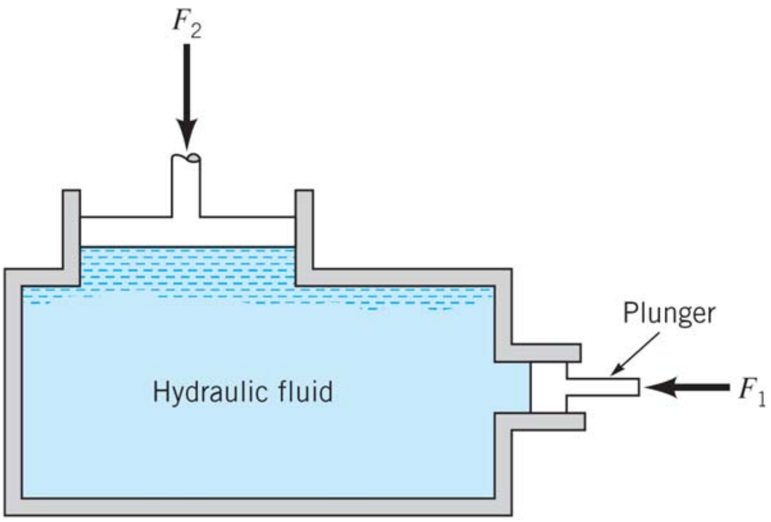
$$\left.\begin{array}{l} p_4 = p_A - 29896 \\ \\ \left.\begin{array}{l} p_5 = p_4 \\ p_6 = p_5 + \rho_w g\times(0.25+0.05) \end{array}\right\} \Rightarrow p_6 = p_4 + 2943 \end{array}\right\} \Rightarrow p_6 = p_A - 29896 + 2943 = p_A - 26953$$

$$\left.\begin{array}{l} p_7 = p_6 \\ p_8 = p_{atm} = p_7 - \rho_w\ g\times 0.475 = p_7 - 63093 \end{array}\right\} \Rightarrow p_8 = p_6 - 63093$$

$$\left.\begin{array}{l} p_8 = p_6 - 63093 \\ p_6 = p_A - 26953 \end{array}\right\} \Rightarrow p_8 = p_A - 26953 - 63093 = p_A - 90046$$

$$\Rightarrow p_A = p_{atm} + 90046 = 101500 + 90046 = 191546 \text{ Pa}$$

4. **The basic elements of a hydraulic press are shown in the figure. The plunger has an area of** $6.4516\ \text{cm}^2$**, and a force,** F_1**, can be applied to the plunger through a lever mechanism having a mechanical advantage of 8 to 1. If the large piston has an area of** $967.74\ \text{cm}^2$**, what load,** F_2**, can be raised by a force of** $133.5\ \text{N}$ **applied to the lever? Neglect the hydrostatic pressure variation.**



Since we are neglecting hydrostatic pressure variation,
the pressure is the same between the planger and the piston.

$$\left.\begin{array}{l} p_1 = p_2 \Rightarrow \dfrac{F_1}{A_1} = \dfrac{F_2}{A_2} \Rightarrow F_2 = F_1 \dfrac{A_2}{A_1} \\ \Rightarrow F_1 = 8 \cdot F_{\text{lever}} \end{array}\right\} \Rightarrow F_2 = 8 \cdot F_{\text{lever}} \dfrac{A_2}{A_1} = 8 \cdot 133.5 \dfrac{967.74}{6.4516} = 160.2 kN$$

5. **Determine the force F_1 acting on the plunger so that the piston would not move (Figure 3). The hydraulic fluid has density $\rho = 850 \text{ kg/m}^3$.**

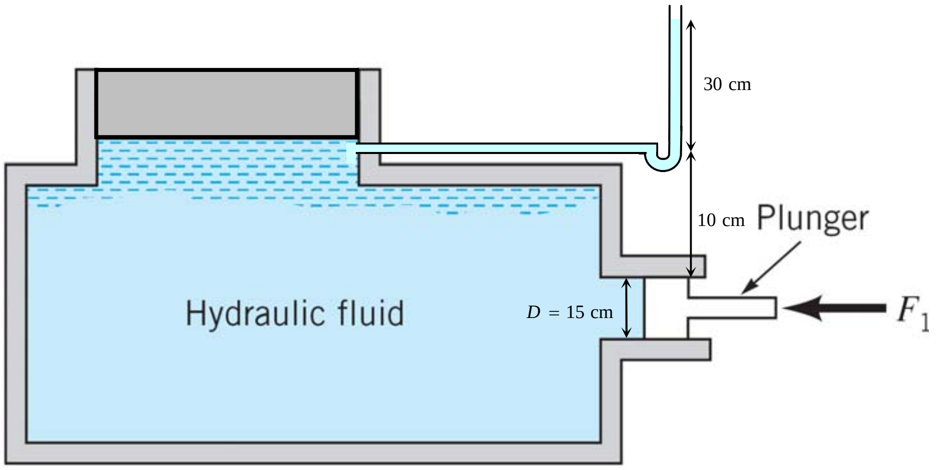


Figure 3: Piston-plunger arrangement

The force acting on the plunger is equal to the pressure at the middle of the plunger multiplied by the area of the plunger:

$$p_{\text{middle of plunger}} = \rho g \underbrace{\left(0.3 + 0.1 + \frac{0.15}{2}\right)}_{h} = 396 \text{ N/m}^2$$

$$\Rightarrow F_1 = p \cdot A = 396 \frac{\pi D^2}{4} = 396 \cdot \frac{0.15^2 \pi}{4} = 70 \text{ N}$$

6. Determine the pressure gage reading and the height h of the mercury manometer if the vapor pressure is $p = 120 \text{ kPa (Abs)}$.

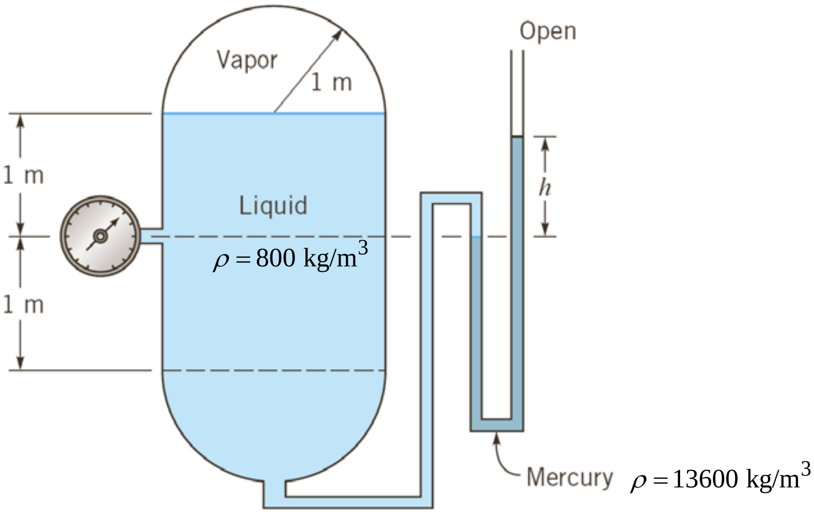
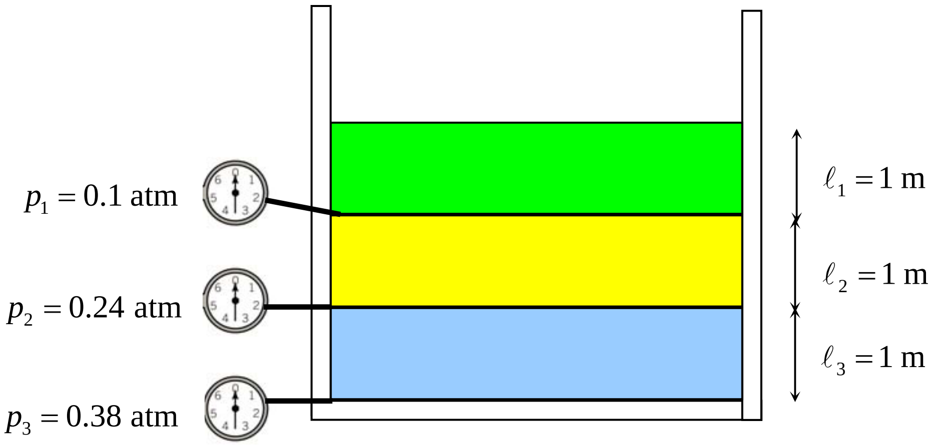


Figure 3: Tank with vapour/liquid (Question3)

Gage pressure $p_g = p_v + \rho_l gh - p_{atm} = 120$ kPa+800*9.81*1-101000=127848-101000 =26848 Pa

$$p_g + 101000 = \rho_m gh + p_{atm} \Rightarrow h = \frac{p_g}{\rho_m g} = \frac{26848}{13600*9.81} = 0.2 \text{ m}$$

7. A container has three different liquids as shown in the figure. Based on the gauge pressures shown on the manometers determine the density of the fluids.



Gage pressure at point 1 = $p_1 = \rho_1 g h_1 = \rho_1 g \ell_1 = 0.1 \text{ atm} = 10132.5 \text{ Pa}$

$$\Rightarrow \rho_1 = \frac{10132.5}{9.81 \cdot 1} = 1033 \ \frac{\text{kg}}{\text{m}^3}$$

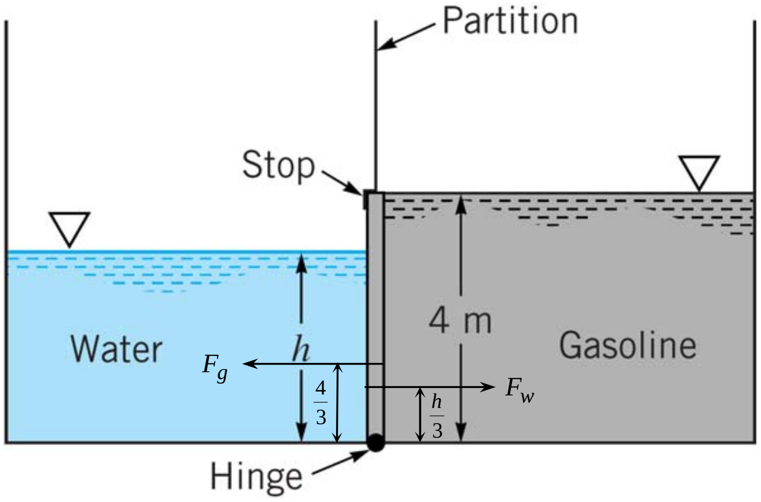
Gage pressure at point 2 = $p_2 = \rho_1 g \ell_1 + \rho_2 g \ell_2 = 10132.5 + \rho_2 g \ell_2 = 0.24 \text{ atm} = 24318 \text{ Pa}$

$$\Rightarrow \rho_2 = \frac{24318\text{-}10132.5}{9.81 \cdot 1} = 1446 \ \frac{\text{kg}}{\text{m}^3}$$

Gage pressure at point 3 = $p_3 = \rho_3 g \ell_3 + 0.24 \text{ atm} = 0.38 \text{ atm}$

$$\Rightarrow \rho_3 = \frac{0.14 \text{ atm}}{9.81 \cdot 1} = \frac{14185.5}{9.81} = 1446 \ \frac{\text{kg}}{\text{m}^3}$$

8. **An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700\ \text{kg/m}^3$ at a depth of 4 m, as shown in the figure. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?**



$F_w = p(middle) \times Area = \rho_w g \dfrac{h}{2}(h \times 2)$ $\qquad F_g = p(middle) \times Area = \rho_g g \dfrac{4}{2}(4 \times 2)$

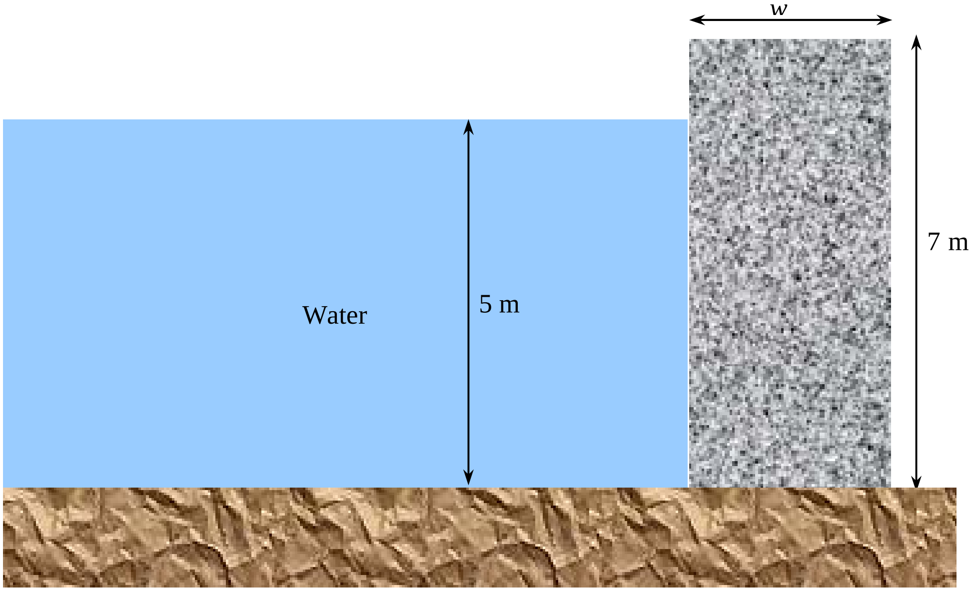
Take moments about the hinge. When the moments are equal the door will open, i.e.

$$F_w \times \frac{h}{3} = F_g \times \frac{4}{3}$$

$$1000 \times 9.81 \times h^2 \times \frac{h}{3} = 700 \times 9.81 \times 4 \times 4 \times \frac{4}{3}$$

$$\Rightarrow h^3 = \frac{700 \times 4^3 \times 3}{1000 \times 3} = 44.8 \Rightarrow h = 3.55\ \text{m}$$

9. Calculate the width of concrete dam that is necessary to prevent the dam from sliding. The specific weight of the concrete is 2400 kg/m^3, and the coefficient of friction between the base of the dam and the foundation is 0.42. Use 1.5 as the factor of safety (F.S.) against sliding. Will it also be safe against overturning? The span of the damn is 1 m.



Water dam

Sliding

Weight of concrete dam=~~V~~ ρg=(1*7*w)*2400*9.81

Sliding force=Hydrostatic force $F_H = \rho_{water} g \frac{5}{2}(5*1) = 122625 \text{ N}$

Sliding resistance= 0.42*165000*w

$$\text{F.S.}=\frac{\text{Sliding resistance}}{\text{Sliding force}} = 1.5 = \frac{69300w}{122625} \Rightarrow w = 2.65$$

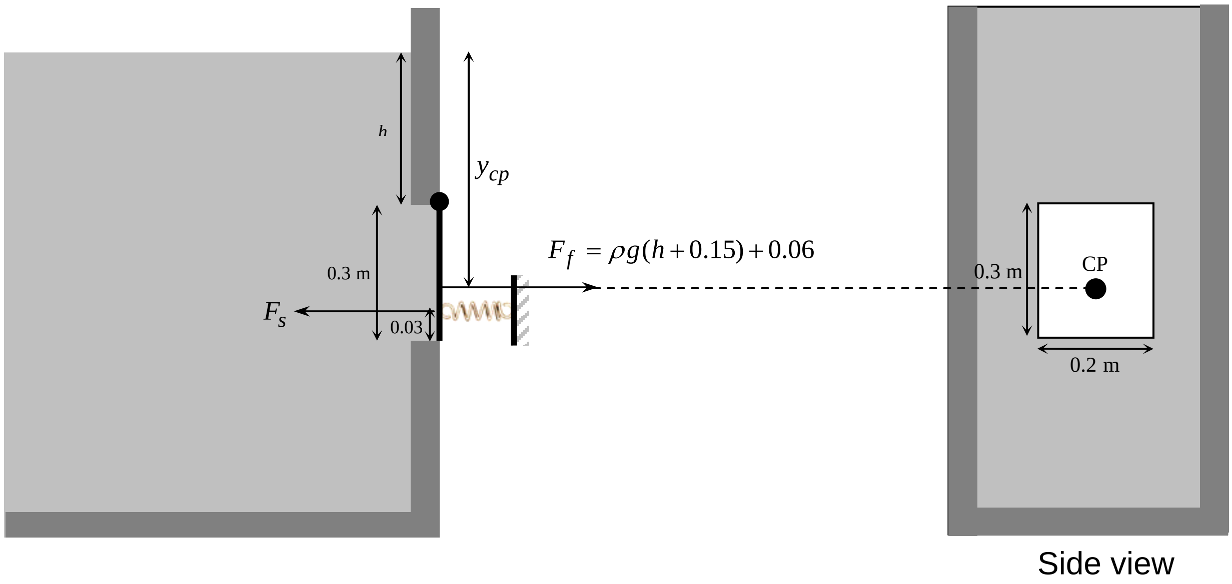
Oveturning

total righting moment=165000*w*w/2

overturning moment=122625*5/3

To be save against oveturning 165000*w*w/2 > 122625*5/3 $\Rightarrow w > \sqrt{\frac{122625*5*2}{3*165000}}$

10. A tank containing fuel of density $850\ \mathrm{kg/m^3}$ has an automatic device to prevent overflow; this consists of a rectangular door hinged at the upper edge closed by a spring. The door is $0.3\ \mathrm{m}$ deep and $0.2\ \mathrm{m}$ wide and the spring exerts a force of $1\ \mathrm{kN}$. Calculate the level of the fuel above the hinge when the door is about to open.



$y_c = h + 0.15$

$$y_R - y_c = \frac{I_{xc}}{y_c A} = \frac{ba^3}{12 y_c ba} = \frac{a^2}{12(h+0.15)} = \frac{0.0075}{h+0.15}$$

The surface of the fuel is at height h above the hinge as shown on the diagram.

The force acting on the door F_g = pressure at the centroid (y_c) $\times$ area of the door

F_g = pressure at the center of the door $\times$ (0.3×0.2)

$F_g = \rho\, g\, y_c \times (0.3\times 0.2) = 850\times 9.81\times (h+0.15)\times 0.06$

$F_g = 500\,(h+0.15)$ N

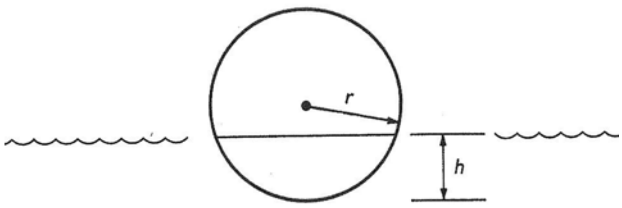
Take moments of the two forces (hydrostatic and spring) about the hinge:

$1000\times(0.3-0.03) = 500\ (h+0.15)\ (y_R - h)$

$$\Rightarrow 1000\times 0.27 = 500(h+0.15)\left(\frac{0.0075}{h+0.15} + h + 0.15 - h\right)$$

$\Rightarrow 270 = 3.75 + 75h + 11.25 \Rightarrow h = 3.4$ m

11. A sphere of density ρ and radius r is placed in a large reservoir filled with a liquid. Find an expression for the depth to which a sphere will sink. If the sphere is displaced from its equilibrium position, find a differential equation that governs its oscillation assuming inviscid flow. Note: the volume of a spherical segment is $\frac{\pi}{3}(3rh^2 - h^3)$.



Archimede's Principle

Buoyancy Force = weight of displaced water = $\frac{\pi}{3}\left(3rh^2 - h^3\right)\rho_w g$

This must be equal to the weight of the sphere = $W = mg = r$ ~~V~~ ρ g = $\frac{4}{3}\pi r^3 \rho g$

So, $\frac{4}{3}\pi r^3 \rho g = \frac{\pi}{3}\left(3rh^2 - h^3\right)\rho_w g \Rightarrow \boxed{3rh^2 - h^3 = 4r^3 \frac{\rho}{\rho_w}}$

Given r, ρ_w and ρ you can solve it to find h.

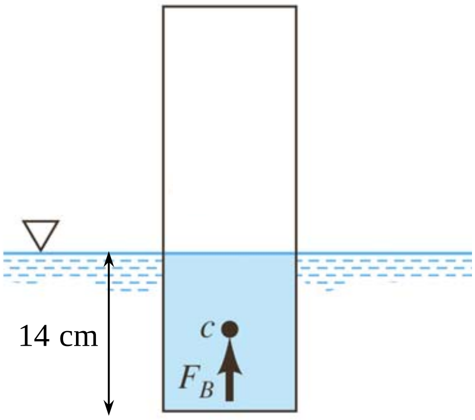
Differential Equation

$F = ma$

$$\Rightarrow \frac{4}{3}\pi r^3 \rho g - \frac{\pi}{3}\left(3rh^2 - h^3\right)\rho_w g = \frac{4}{3}\pi r^3 \rho \frac{\mathrm{d}^2 h}{\mathrm{d}t^2}$$

$$\Rightarrow g - \frac{g}{4r^3}\left(3rh^2 - h^3\right)\frac{\rho_w}{\rho} = \frac{\mathrm{d}^2 h}{\mathrm{d}t^2}$$

12. **A solid cylinder of length 0.3 m and diameter 10 cm is floating in water (see figure). Determine:**
   a. **The upward hydrostatic force (F_B) acting on the bottom of the cylinder.**
   b. **The density of the cylinder.**



a. $F_b = \forall_{\substack{\text{displaced}\\ \text{liquid}}} \rho_{fluid}\, g = \frac{\pi D^2}{4} 0.14 \cdot 1000 \cdot 9.81 = 10.78\ \text{N}$

b. Because the cylinder is not moving $\sum \text{F} = 0 = F_b - W = 0$

where W is the weight of the cylinder.

$\Rightarrow 10.78 = \forall_{\text{cylinder}}\, \rho_{\text{cylinder}}\, g \Rightarrow \rho_{\text{cylinder}} = \frac{10.78}{\forall_{\text{cylinder}}\, g} = \frac{10.78}{\frac{\pi D^2}{4} 0.3 \cdot 9.81} = 466.4\ \frac{\text{kg}}{\text{m}^3}$

13. **A horizontal gate, 1.5 meters wide, is held in place by a cable as shown in the Figure. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.**

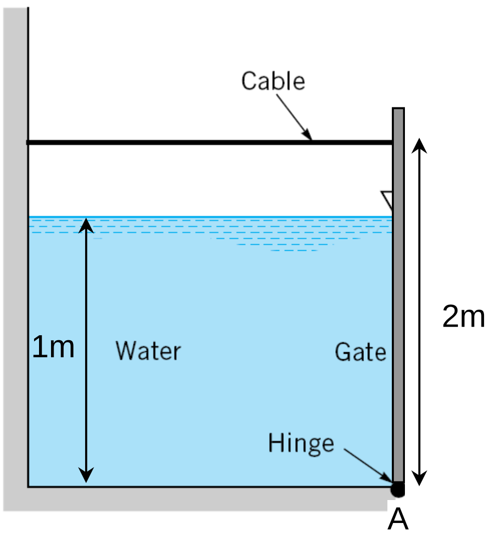


Figure: Tank with a gate held by a cable