

**Homework Assignment on
Basic Units and Viscosity**

1. Complete the following table

Property/ Quantity	Definition in Greek (optional)	Symbol	Units/ definition	Equation
distance	απόσταση	x	m	
angle	γωνιά	θ	dimensionless	$\frac{\text{arc length}}{\text{radius}}$
perimeter	περίμετρος	P	m	rectangle = $2(a + b)$ circle = $2\pi R = \pi D$
area	Εμβαδό	A	m^2	rectangle = $a \times b$ circle = $\pi R^2 = \pi \frac{D^2}{4}$
Volume	όγκος	\forall	m^3	cube = a^3 rectangular prism = $\ell \times w \times h$ cylinder = $\pi R^2 h$ sphere = $\frac{4}{3} \pi R^3$
mass	Μάζα	m	kg	NA
density	πυκνότητα	ρ	$\frac{\text{kg}}{\text{m}^3}$	$\frac{m}{\forall}$
velocity	ταχύτητα	V, u	m/s	$\frac{s}{t}, \frac{dx}{dt}$
Angular velocity	Γωνιακή ταχύτητα	ω	1/s	$\frac{d\theta}{dt}$
acceleration	επιτάχυνση	a	$\frac{\text{m}}{\text{s}^2}$	$\frac{\delta U}{\delta t}$

Force	δύναμης	F	$N \equiv kg \times m/s^2$	$F = m a$
Torque	ροπή	T	$N \cdot m$	$F \cdot s$
viscosity	ιξώδες	μ	$\frac{kg}{m \cdot s}$	
Shear stress	Διατμητική Τάση	τ	$\frac{N}{m^2}$	$\frac{F_{shear}}{A}$, $\mu \frac{du}{dy}$ (for Newtonian fluids)
Pressure	Πίεση	p	$\frac{N}{m^2} = Pa$	$\frac{F_n}{A}$
Pressure Head		h or H	m	$h = \frac{P}{\rho g}$
Energy Work	Ενέργεια Έργο	W, E	J	$F \cdot s, mgh, \frac{1}{2} mu^2$
Power	Ισχύς	P	W	$W / t, F \cdot u, T \cdot \omega$
Reynolds number	Αριθμός Reynolds	Re	dimensionless	$\frac{\rho u D}{\mu} = \frac{u D}{\nu}$
Volumetric Flow Rate	Ογκομετρική παροχή	\dot{V} or Q	m^3 / s	$\dot{V} = uA$ where $\left\{ \begin{array}{l} A \equiv \text{x-sectional area,} \\ \text{component of velocity} \\ u \equiv \text{normal to the} \\ \text{x-sectional area} \end{array} \right.$ Assumptions: 1. Uniform Flow 2. Constant Density
Mass Flow Rate	Παροχή μάζας	\dot{m}	kg / s	$\dot{m} = \rho \dot{V}$ where $\left\{ \begin{array}{l} \rho \equiv \text{density} \\ \dot{V} \equiv \text{see above} \end{array} \right.$

2. The **Reynolds number (Re)** is defined as:

$$\text{Re} = \frac{\rho VD}{\mu},$$

where ρ is the density, V is the velocity, D is length and μ is the viscosity.

a. Determine its units

b. Express it in terms of the dynamic viscosity (ν).

c. A Newtonian fluid having a viscosity of $\mu = 0.38 \text{ N}\cdot\text{s}/\text{m}^2$ and $\rho = 910 \text{ kg}/\text{m}^3$ flows through a pipe with diameter $D = 25 \text{ mm}$ and velocity of $2.6 \text{ m}/\text{s}$. Determine the value of the Reynolds number.

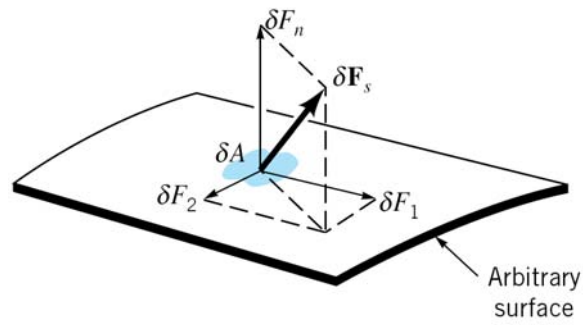
$$a. \text{Re} = \frac{\rho VD}{\mu} = \frac{\frac{\text{kg}}{\text{m}^3} \frac{\text{m}}{\text{s}} \text{m}}{\frac{\text{kg}}{\text{m}\cdot\text{s}}} = \text{unitless, Reynolds number has no units, i.e. dimensionless}$$

$$b. \nu = \frac{\mu}{\rho} \Rightarrow \mu = \nu\rho$$

$$\text{Substitute in expression for Reynolds number } \text{Re} = \frac{\rho VD}{\nu\rho} = \frac{VD}{\nu}$$

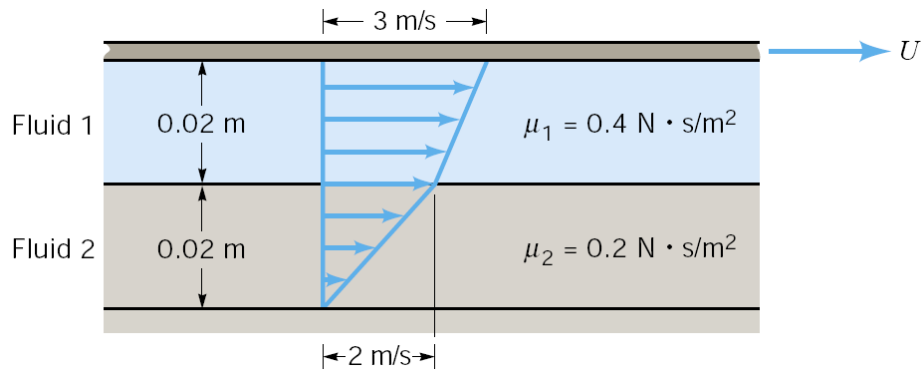
$$c. \text{Re} = \frac{\rho VD}{\mu} = \frac{910 \cdot 2.6 \cdot 0.025}{0.38} = 156$$

3. The Figure shows a small element of surface area of an airfoil.



Write the expression for the normal stress.	$\frac{\delta F_n}{\delta A}$
Write the expression for the shear stress.	$\frac{\delta F_1}{\delta A}$ or $\frac{\delta F_2}{\delta A}$
Write the expression for the pressure acting on the surface.	$\frac{\delta F_n}{\delta A}$
What kind of stresses do you expect to observe in a static fluid?	Only normal stresses
Write the expression for the shear stress in a Newtonian fluid.	$\tau = \mu \frac{du}{dy}$

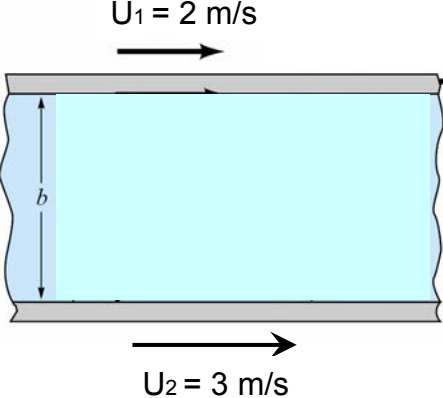
4. Let two layers of fluid be dragged along by the motion of an upper plate as shown in the Figure. The bottom plate is stationary. The top fluid puts a shear stress on the upper plate, and the lower fluid puts a shear stress on the bottom plate. Determine the ratio of these two shear stresses.



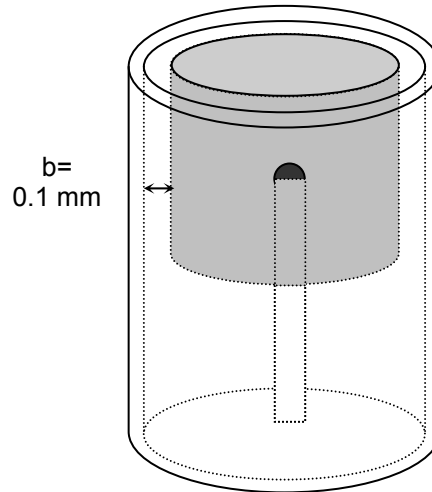
Two layers of fluid in a rectangular channel

$$\left. \begin{aligned} \tau_2 &= \mu_2 \frac{du_2}{dy} = 0.2 \frac{2-0}{0.02} = 20 \\ \tau_1 &= \mu_1 \frac{du_1}{dy} = 0.4 \frac{3-2}{0.02} = 20 \end{aligned} \right\} \Rightarrow \tau_2 = \tau_1$$

5. Consider two parallel plates filled with oil with viscosity $\mu = 0.5 \text{ N}\cdot\text{s}/\text{m}^2$ and moving with the velocities indicated. Assume the velocity distribution in the gap to be linear. Find the shear stress.



6. A piston with diameter $D = 50 \text{ mm}$ and length $l = 75 \text{ mm}$ long, moves vertically in an open-ended lubricated cylinder with a radial clearance of 0.1 mm . When falling due to its own weight, the piston moves through 30 mm in 4.2 s at a steady velocity. When a mass of 0.05 kg is added to the piston, it moves with steady velocity through the same distance in 2.4 s . Calculate the viscosity of the oil and the mass of the piston.



- a. If the piston is moving with velocity V , then the fluid within the radial clearance will experience shearing stress. The layer next to the piston will move with velocity V and the layer next to the cylinder will not move.

$$\text{shearing stress } \tau = \mu \frac{du}{dy} \approx \mu \frac{\delta u}{\delta y} = \mu \frac{V - 0}{b} = \mu \frac{V}{b}$$

- b. The shear stress is also equal to $\tau = \frac{F}{A}$, where F is the downwards force and A the area of the curved surface of the cylinder:

$$\tau = \frac{mg}{2\pi rh}$$

$$\text{Hence } \boxed{\frac{mg}{2\pi rh} = \mu \frac{V}{b}}$$

Case 1

Cylinder moves due to its own weight

$$V = \frac{\delta x}{\delta t} = \frac{0.03}{4.2} = 0.007143 \text{ m/s}$$

$$\frac{m \cdot 9.81}{\pi \cdot 0.05 \cdot 0.075} = \mu \frac{0.007143}{0.0001} \Rightarrow \boxed{m = 0.0858\mu}$$

Case 2

Cylinder moves due to an additional weight

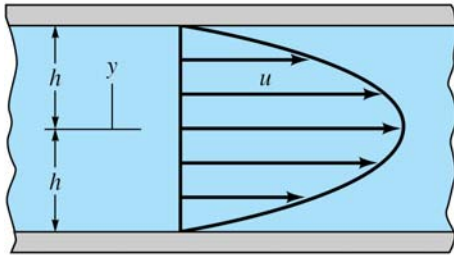
$$V = \frac{\delta x}{\delta t} = \frac{0.03}{2.4} = 0.0125 \text{ m/s}$$

$$\frac{(m + 0.05) \cdot 9.81}{0.0118} = \mu \frac{0.0125}{0.001} \Rightarrow \boxed{m + 0.05 = 0.1502\mu}$$

Combine the last two equations

$$0.0858\mu + 0.05 = 0.1502\mu \Rightarrow \mu = 0.777 \frac{\text{kg}}{\text{ms}}$$

7. The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates is given by the equation



$$u(y) = \frac{3V}{2} \left(1 - \left(\frac{y}{h} \right)^2 \right)$$

- d. Which point corresponds to the velocity V ?
 e. What is the maximum velocity?
 f. If the fluid is water determine the shear stress at the wall.

- a. To find which point corresponds to velocity V

set $u = V$

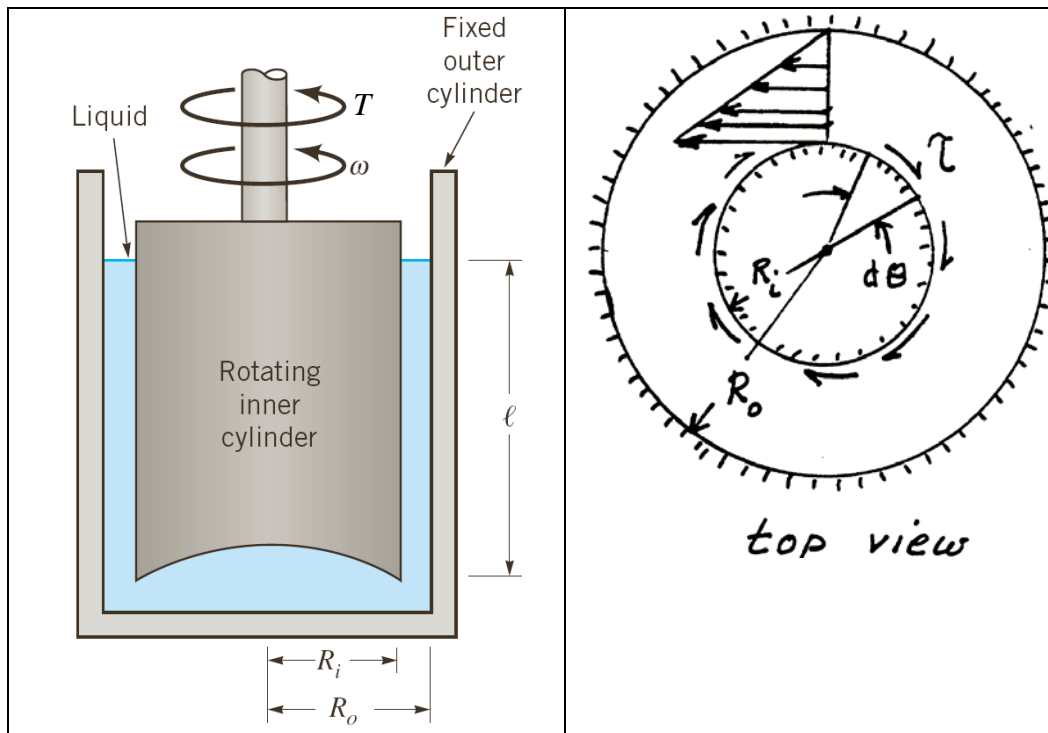
$$V = \frac{3V}{2} \left(1 - \left(\frac{y}{h} \right)^2 \right) \Rightarrow \left(\frac{y}{h} \right)^2 = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow y = \sqrt{\frac{1}{3}}h$$

- b. To find the maximum velocity differentiate $\frac{du}{dy}$ and set to zero

$$\frac{du}{dy} = -\frac{3Vy}{h^2} = 0 \Rightarrow y = 0 \text{ in agreement with the velocity profile.}$$

c. $\tau = \mu \frac{du}{dy} = -\mu \frac{3Vy}{h^2}$

8. The viscosity of liquids can be measured through the use of a *rotating cylinder viscometer* of the type illustrated in the figure. In this device the outer cylinder is fixed and the inner cylinder is rotated with an angular velocity, ω . The torque T required to develop ω is measured and the viscosity is calculated from these two measurements. If the liquid under testing is water determine the torque developed if $\omega = 200$ rpm, $\ell = 15$ cm, $R_o = 7$ cm, $R_i = 6.5$ cm. Neglect end effects and assume the velocity distribution in the gap is linear.



If the cylinder is rotating at constant speed it implies that there is a balance of forces (torques). So, the torque acting on the shaft must balance the torque due to the shear stress acting on the surface of the cylinder. This force

$$\text{is equal to } \tau = \mu \frac{du}{dr} \approx \mu \frac{\delta u}{\delta r} = \mu \frac{V_{\text{inner cylinder}} - V_{\text{outer cylinder}}}{r_{\text{inner}} - r_{\text{outer}}} = \mu \frac{\omega R_i - 0}{R_i - R_o} = \mu \frac{\omega R_i}{R_i - R_o}$$

The total force acting on the cylinder is equal to the shear stress multiplied by the outer area of the cylinder

$$F = \tau A = \mu \frac{\omega R_i}{R_o - R_i} 2\pi R_i \ell$$

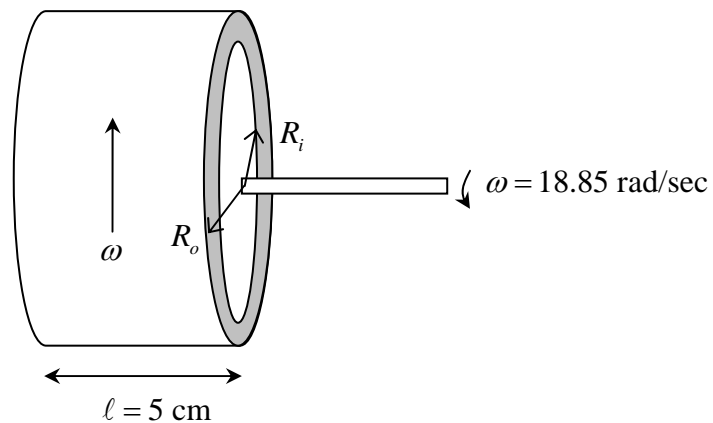
The torque due to this the force (the moment arm in this case is R_i) is

$$T = FR_i = \mu \frac{\omega R_i}{R_o - R_i} 2\pi R_i \ell R_i = 0.00115 \frac{200 \cdot 2\pi}{60} \frac{0.065}{0.07 - 0.065} 2\pi \cdot 0.065^2 \cdot 0.15 = 0.0012 \text{ Nm}$$

$$P = T \times \omega = 0.0012 \frac{200 \cdot 2\pi}{60}$$

9. Consider two concentric cylinders filled with oil with viscosity $\mu = 0.5 \text{ N}\cdot\text{s}/\text{m}^2$. The inner cylinder has radius of $R_i = 7.6 \text{ cm}$ and the gap width between cylinders is $R_o - R_i = 2.5 \text{ mm}$. The outer cylinder is rotating with $\omega = 18.85 \text{ rad/s}$. If the inner cylinder is rotating in the opposite direction with the same angular velocity determine:
- The shear stress on the inner cylinder
 - The torque required.
 - The power required.

Assume the velocity distribution in the gap to be linear.



Similar to the previous exercise the shear stress

$$\text{is equal to } \tau = \mu \frac{du}{dr} \approx \mu \frac{\delta u}{\delta r} = \mu \frac{V_{\text{inner cylinder}} - V_{\text{outer cylinder}}}{r_{\text{inner}} - r_{\text{outer}}} = \mu \frac{\omega R_i + \omega R_o}{R_i - R_o} = \mu \frac{\omega (R_i + R_o)}{R_i - R_o}$$

The total force acting on the cylinder is equal to the shear stress multiplied by the outer area of the cylinder

$$F = \tau A = \mu \frac{\omega (R_i + R_o)}{R_i - R_o} 2\pi R_o \ell$$

The torque due to this the force (the moment arm in this case is R_i) is

$$T = FR_i = \mu \frac{\omega (R_i + R_o)}{R_i - R_o} 2\pi R_o \ell R_i$$

$$P = T \times \omega$$