



HOMWORK ASSIGNMENT ON MASS CONSERVATION

1. Water is flowing through a pipe of diameter 0.5 m. The volumetric flow rate is 3.5 m³/s. Find the mass flow rate, the fluid velocity, and the Reynolds number.

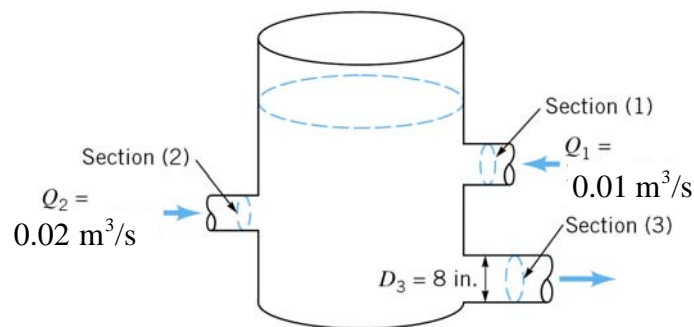
$$\dot{V} = Q = uA = \frac{\dot{m}}{\rho} \Rightarrow \dot{m} = \rho Q = 1000 \times 3.5 = 3500 \frac{\text{kg}}{\text{s}}$$

$$\left. \begin{array}{l} u = \frac{Q}{A} \\ A = \frac{\pi D^2}{4} \end{array} \right\} \Rightarrow u = \frac{3.5}{\frac{\pi \cdot 0.5^2}{4}} = 17.83 \frac{\text{m}}{\text{s}}$$

$$\text{momentum flux } (\dot{M}) = \dot{m}u = 3500 \cdot 17.83 = 62.4 \text{ kN}$$

$$\text{Re} = \frac{\rho u D}{\mu} = \frac{1000 \times 17.83 \times 0.5}{1.12 \times 10^{-3}} = 8.0 \times 10^6$$

2. Water enters a cylindrical tank through two pipes at rates of 0.02 m³/s and 0.01 m³/s. If the level of the water in the tank remains constant, calculate the average velocity of the flow leaving the tank through the 8 in inside diameter pipe at section 3.



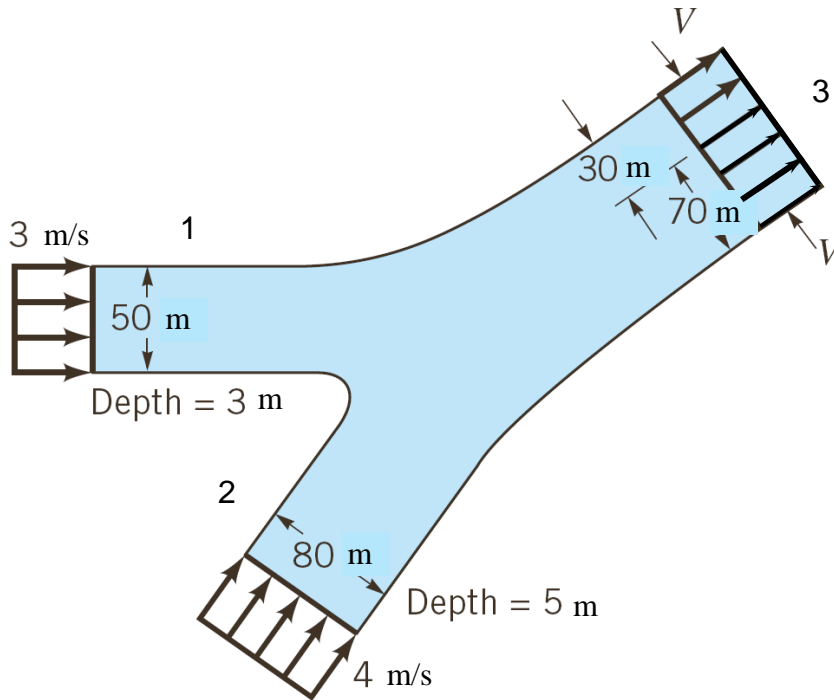
Mass Conservation

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \rho_1 Q_1 + \rho_2 Q_2 = \rho_3 Q_3$$

$$\text{incompressible} \Rightarrow \rho_1 = \rho_2 = \rho_3 \Rightarrow Q_1 + Q_2 = Q_3 = 0.02 + 0.01 = 0.03 \text{ m}^3/\text{s}$$

$$u_3 = \frac{Q_3}{A_3} = \frac{0.03}{\frac{\pi D_3^2}{4}} = \frac{0.03}{\frac{\pi (8 \cdot 0.0254)^2}{4}} = 0.93 \text{ m/s}$$

3. Two rivers merge to form a larger river as shown in the figure. At a location downstream from the junction the two streams completely merge and the depth is 6 m. Determine the value of V .



Mass Conservation

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

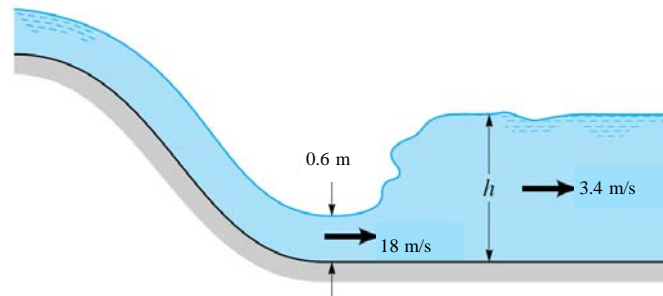
$$\Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\Rightarrow \rho_1 u_1 A_1 + \rho_2 u_2 A_2 = \rho_3 u_3 A_3$$

$$\Rightarrow u_1 A_1 + u_2 A_2 = u_3 A_3$$

$$\Rightarrow 3 \cdot 50 \cdot 3 + 4 \cdot 80 \cdot 5 = V \cdot 100 \cdot 6 \Rightarrow V = \frac{3 \cdot 50 \cdot 3 + 4 \cdot 80 \cdot 5}{600} = 3.17 \text{ m/s}$$

4. A hydraulic jump is in place downstream from a spill-way as indicated in the figure. Upstream of the jump, the depth of the stream is 0.6 m and the average stream velocity is 18 m/s. Just downstream of the jump, the average stream velocity is 3.4 m/s. Calculate the depth of the stream, h , just downstream of the jump, if the width of the stream is the same throughout.



If we assume that the width (b) is the same then:

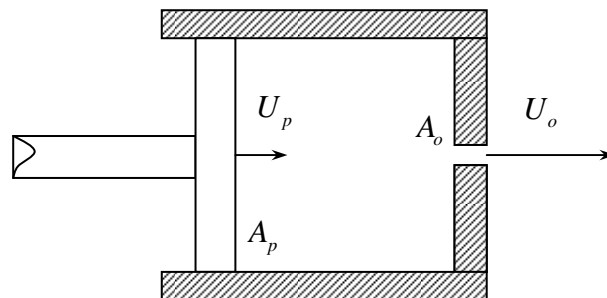
Mass Conservation

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\Rightarrow \rho_{in} u_{in} A_{in} = \rho_{out} u_{out} A_{out} \Rightarrow \rho \cdot 18 \cdot b \cdot 0.6 = \rho \cdot 3.4 \cdot b \cdot h$$

$$\Rightarrow h = \frac{18 \cdot 0.6}{3.4} = 3.1765 \text{ m}$$

5. A piston with cross-sectional area A_p is moving with constant velocity U_p . Determine the relation between A_p , U_p and A_o , U_o if the fluid inside the cylinder is incompressible.



The amount of mass pushed by the piston must exit from the small hole at the other end. Assuming of course that the cylinder is full of liquid.

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\Rightarrow \rho_{in} u_{in} A_{in} = \rho_{out} u_{out} A_{out} \Rightarrow U_p \cdot A_p = U_o \cdot A_o$$