FREDERICK INSTITUTE OF TECHNOLOGY

Mechanical Engineering Programme of Study

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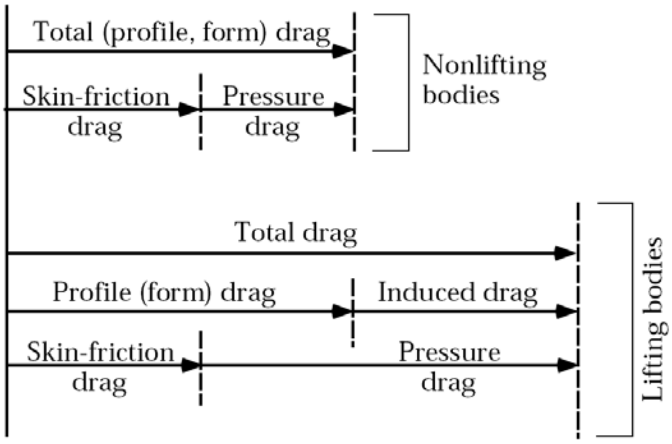
Fluid Mechanics

Homework Assignment Solution

1. State briefly the forces associated with a lifting and a non-lifting body. Describe their physical origin. State the non-dimensional coefficients associated with drag and lift.

The total drag acting on a body can be regarded as the sum of several components.



In general, the drag is composed of a frictional component, related to viscous shearing in boundary layers, and pressure drag, which is related to the pressure differential between the fore and aft of the body. If the body also experiences lift, then there is an additional component of the pressure drag, called induced drag. The drag force is typically written in terms of the non-dimensional drag coefficient:

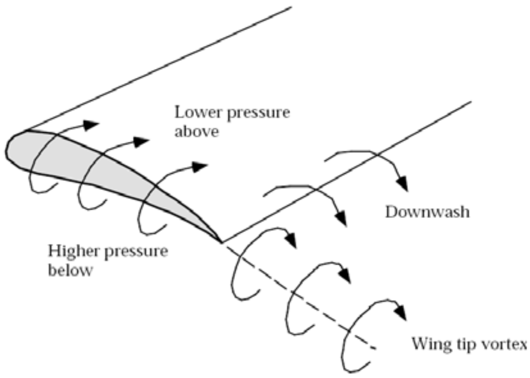
$$C_D = \frac{\text{Drag Force}}{\frac{1}{2}\rho U^2 A}$$

where ρ is the density of the fluid, U is the velocity of the fluid and A is the reference area. The reference area is usually the projected area of the body in the direction of flow. For example, the reference area for a sphere would be πr^2, where 'r' is the radius. In the case of a circular cylinder, the area would be $d \cdot L$, where L is the length. Sometimes, however, a different reference area is used. The lift coefficient is defined in the same manner as the drag coefficient:

$$C_D = \frac{\text{Lift Force}}{\frac{1}{2}\rho U^2 A}.$$

Induced Drag

In practice, all lifting bodies have finite length. Near the end of body, the pressure differential between the bottom and top surfaces produces a flow of air around the end, from beneath the body to the top of the body. This flow produces a vortex, the so called wing tip vortex, which then trails off into the wake. The wing tip vortex and the corresponding downwash near the wing tip acts to reduce the lift on the wing and increase the drag. The additional drag is termed induced drag.



2. **For small Reynolds number $\mathrm{Re} < 1$, the drag coefficient for a non-lifting sphere is approximated by $C_D = 24/\mathrm{Re}$. A small gas bubble of diameter 10 microns is moving slowly in water. What is the maximum velocity that allows the use of the above equation? What is the drag force under these conditions?**

The maximum velocity is obtained by setting the Reynolds number $\mathrm{Re} = 1$.

$$\mathrm{Re} = \frac{\rho U D}{\mu} = 1 \Rightarrow \frac{1000 \cdot U \cdot 10^{-6}}{1.12 * 10^{-3}} = 1$$

$$\Rightarrow U = \frac{1.12 * 10^{-3}}{1000 * 10^{-6}} = 1.12 \text{ m/s}$$

$$\mathrm{C}_D = \frac{\text{Drag Force}}{\frac{1}{2}\rho U^2 A} = \frac{24}{\underbrace{\mathrm{Re}}_{=1}}$$

$$\Rightarrow \text{Drag Force} = \frac{1}{2}\ 24 \rho U^2 A$$

$$= 12 \cdot 1000 \cdot 1.12^2 \cdot \pi r^2 = 1.1822 * 10^{-6}\ \mathrm{N}$$

3. **When a model aircraft is tested, the size of the model is naturally less than that of the prototype and the model is tested in the same fluid (atmospheric air) as is used for the prototype. If the model is constructed to one-fifth scale and the prototype is intended to fly at $300 km/h$ at what air velocity the model should be tested? Do you think that the results between the prototype and the model would be equivalent? What other factors have to be taken into consideration?**

The usual processes of dimensional analysis yield the result:

$$C_D = \frac{\text{Drag Force}}{\frac{1}{2}\rho U^2 A} = f(\text{Re}).$$

This equation is true for both model and prototype. Hence, for true dynamic similarity between model and prototype, the Reynolds number must be the same. Consequently, the function $f(\text{Re})$ has the same value for both prototype and model and so $C_D = \frac{\text{Drag Force}}{0.5\rho U^2 A}$ is the same in each case. Using suffix m for the model and p for prototype, we may write

$$\frac{F_p}{0.5\rho_p U_p{}^2 A_p} = \frac{F_m}{0.5\rho_m U_m{}^2 A_m}$$

This result is only valid only if the test on the model is carried out under such conditions that the Reynolds number is the same as for the prototype. Then

$$\text{Re}_m = \text{Re}_p \Rightarrow \frac{\rho_m U_m l_m}{\mu_m} = \frac{\rho_p U_p l_p}{\mu_p} \text{ and so}$$

$$U_m = U_p \left(\frac{l_p}{l_m}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right) = U_p (5)(1)(1) = 1500 \text{ km/hr}$$

This velocity is the *corresponding velocity.* Only when the model is tested at the corresponding velocity does the formation of eddies and so on take place at points corresponding to those on the prototype, and only then are the overall flow patterns exactly similar.
However, at velocities as high as 1500 km/hr, the effects of the compressibility of air become important and the pattern of flow round the model will be quite different from that round the prototype, even though the Reynolds number is kept the same.

4. **An aircraft is to fly at a height of $9km$ (where the temperature and pressure are -45^oC and $30.2kPa$ respectively) at $400m/s$. A $1/20th$-scale model is tested in a pressurized wind-tunnel in which the air is at 15^oC. For complete dynamic similarity what pressure and velocity should be used in the wind-tunnel? (For air at TK, $\mu \propto T^{3/2}/(T+117)$.)**

For dynamic similarity in cases where the effects of compressibility are important for the prototype, the Mach numbers also must be identical. For complete similarity, therefore, we require

$$\mathrm{Re}_m = \mathrm{Re}_p \Rightarrow \frac{\rho_m u_m l_m}{\mu_m} = \frac{\rho_p u_p l_p}{\mu_p} \text{ (equality of Reynolds number)} \quad \text{... Eq. 1}$$

$$\frac{u_m}{a_m} = \frac{u_p}{a_p} \text{ (equality of Mach number)} \quad \text{... Eq. 2}$$

where $a \equiv \sqrt{\gamma RT}$ (speed of sound). ... Eq. 3

From eq. 2 we get $u_m = u_p \dfrac{a_m}{a_p} = u_p \dfrac{\sqrt{\gamma RT_m}}{\sqrt{\gamma RT_p}} = u_p \sqrt{\dfrac{T_m}{T_p}}$

$$= 400\sqrt{\frac{(15+273.15)}{(-45+273.15)}} = 449.53\text{m/s}$$

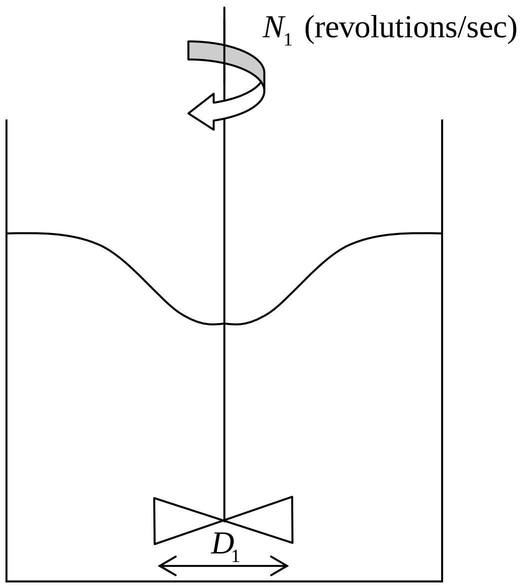
Substituting the equation of the ideal gas law $p = \rho RT$ in Eq. 1

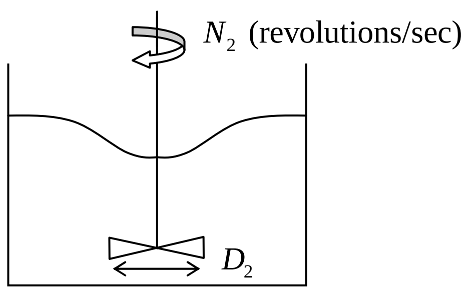
$$\frac{\left.\frac{p}{RT}\right)_m u_m l_m}{\mu_m} = \frac{\left.\frac{p}{RT}\right)_p u_p l_p}{\mu_p} \Rightarrow p_m = p_p \frac{\mu_m}{\mu_p}\frac{T_m}{T_p}\frac{u_p}{u_m}\frac{l_p}{l_m} = p_p \frac{\dfrac{T_m^{3/2}}{(T_m+117)}}{\dfrac{T_p^{3/2}}{(T_p+117)}}\frac{T_m}{T_p}\frac{u_p}{u_m}\frac{l_p}{l_m}$$

$$= p_p \frac{(T_p+117)}{(T_m+117)}\frac{T_m^{5/2}}{T_p^{5/2}}\frac{u_p}{u_m}\frac{l_p}{l_m} =$$

$$= 30.2\frac{(-45+273.15+117)}{(15+273.15+117)}\frac{(15+273.15)^{5/2}}{(-455+273.15)^{5/2}}\frac{400}{449.53}20 = 820.8 \text{ kPa}$$

5. **It is desired to predict vortex depth for steady-state flow in a large unbaffled tank of oil (left figure) as a function of agitator speed (**N revolutions/sec**). It is proposed to do this by means of a model study in a smaller geometrically similar tank (right figure):**
   i. Determine the Reynolds and Froude numbers relevant for this flow,
   ii. If the smaller tank is to have half the linear dimensions of the larger tank, determine the kinematic viscosity of the fluid used, so that the two vortices are similar.





$$\mathrm{Re} = \frac{\rho u l}{\mu} \text{ and } \mathrm{Fr}=\sqrt{\frac{u^2}{\mathrm{g}\cdot h}}.$$

In this flow a typical velocity $u = \pi DN$ which is the velocity of the tip of the agitator. Hence the dimensionless numbers become

$$\mathrm{Re} = \frac{\rho \pi DND}{\mu} = \frac{\rho \pi ND^2}{\mu} \text{ and } \mathrm{Fr} = \sqrt{\frac{(\pi DN)^2}{\mathrm{g}\cdot D}} = \sqrt{\frac{\pi^2 DN^2}{\mathrm{g}}}$$

For similarity

$$\left.\begin{aligned} &\mathrm{Fr}_1 = \mathrm{Fr}_2 \Rightarrow \sqrt{\frac{\pi^2 D_1 N_1^2}{\mathrm{g}}} = \sqrt{\frac{\pi^2 D_2 N_2^2}{\mathrm{g}}} \Rightarrow \frac{N_2}{N_1} = \sqrt{\frac{D_1}{D_2}} \\ &\mathrm{Re}_1 = \mathrm{Re}_2 \Rightarrow \frac{\rho_1 \pi N_1 D_1^2}{\mu_1} = \frac{\rho_2 \pi N_2 D_2^2}{\mu_2} \Rightarrow \frac{N_2}{N_1} = \frac{\nu_2}{\nu_1}\left(\frac{D_1}{D_2}\right)^2 \end{aligned}\right\} \Rightarrow$$

$$\sqrt{\frac{D_1}{D_2}} = \frac{\nu_2}{\nu_1}\left(\frac{D_1}{D_2}\right)^2 \Rightarrow \frac{\nu_2}{\nu_1} = \left(\frac{D_2}{D_1}\right)^{(3/2)}$$

6. **The drag on a stationary hemispherical shell with its open, concave side towards an oncoming air stream is to be investigated by experiments on a half-scale model in water. For a steady air velocity of 30 m/s determine the corresponding velocity of the water relative to the model, and the drag on the prototype shell if that on the model is 152 N.**

In order to have similarity the Reynolds numbers between the prototype and the model must the same:

$$\mathrm{Re}_p = \mathrm{Re}_m \Rightarrow \frac{\rho_p u_p D_p}{\mu_p} = \frac{\rho_m u_m D_m}{\mu_m}$$

$$u_m = \frac{\rho_p}{\rho_m}\frac{D_p}{D_m}\frac{\mu_m}{\mu_p} u_p$$

$$= \frac{1.225}{1000} 2 \frac{1.12\cdot 10^{-3}}{1.79\cdot 10^{-5}} 30 = 4.6 \text{ m/s}$$

The dimensionless drag is given by

$$\frac{F_m}{\rho_m D_m^2 u_m^2} = \frac{F_p}{\rho_p D_p^2 u_p^2} \Rightarrow F_p = \frac{152}{\frac{1000}{1.225}\cdot\frac{1}{4}\cdot\left(\frac{4.6}{30}\right)^2} = \frac{152}{4.79} = 31.7 \text{ N}$$