FREDERICK UNIVERSITY CYPRUS

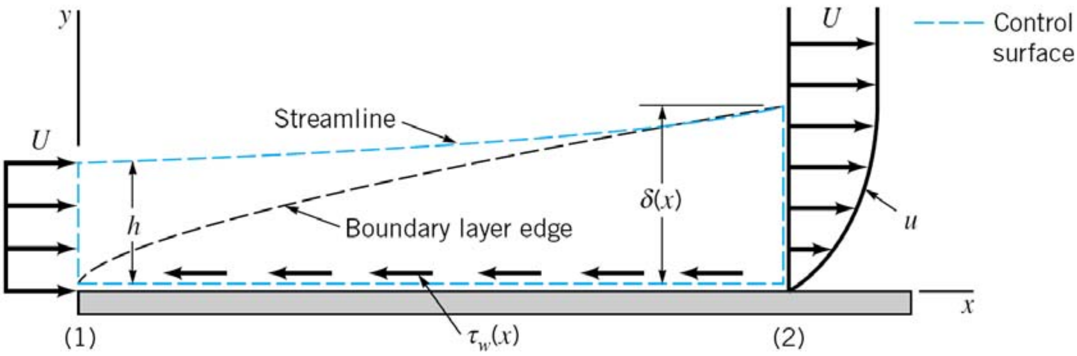
Department of Mechanical Engineering

AMEE401 / AUTO400 **Instructor: Marios M. Fyrillas**
Aerodynamics **Email: eng.fm@fit.ac.cy**

Homework Assignment #4

QUESTION 1

Consider the boundary layer flow on a flat plate of width b (shown on the Figure below).



a. **Using mass conservation show that** $Uh = \int_0^{\delta[x]} u[x,y]\,\mathrm{d}y.$

Page 554 in textbook

b. **Using the momentum equation show that the Drag force on the plate is given by**

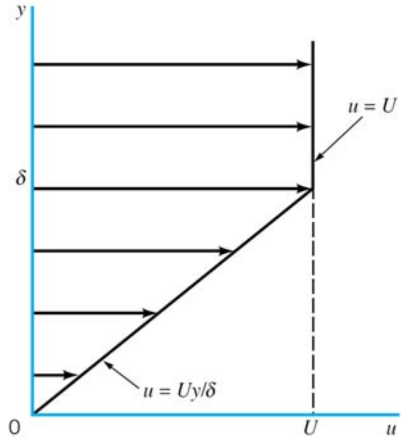
$$D = \rho b \int_0^{\delta[x]} u[x,y]\,(U - u[x,y])\,\mathrm{d}y.$$

Page 554 in textbook

c. **Employing the shear stress on the wall show that the Drag force can be also obtained through** $D = b\int_0^x \tau_w[\omega]\,\mathrm{d}\omega,$ **where** $\tau_w[x] = \mu \dfrac{\partial u}{\partial y}(x, y=0).$

Page 553 in textbook (Eq. 9.19)

d. **Assuming that the velocity profile is approximated as** $u = U\,y/\delta[x]$ **for** $0 \le y \le \delta[x]$ **and** $u = U$ **for** $y > \delta[x]$ **as shown in the Figure below, determine the shear stress.**



x

From (b) we obtain that

$$D = \rho b \int_0^{\delta[x]} \frac{U\,y}{\delta[x]}\left(U - \frac{U\,y}{\delta[x]}\right) \mathrm{d}y = \frac{1}{6}\rho b U^2 \delta[x]$$

$$\Rightarrow \frac{\mathrm{d}D[x]}{\mathrm{d}x} = \frac{1}{6}\rho b U^2 \frac{\mathrm{d}\delta[x]}{\mathrm{d}x}$$

From (c) we obtain that

$$\frac{\mathrm{d}D}{\mathrm{d}x} = b\ \tau_w[x] = b\mu \frac{\partial u}{\partial y}(x, y=0) = b\mu \frac{U}{\delta[x]}$$

Equating above two expressions we get

$$\frac{1}{6}\rho b U^2 \frac{\mathrm{d}\delta[x]}{\mathrm{d}x} = b\mu \frac{U}{\delta[x]}$$

Solving this differential equation we obtain

$$\delta[x] = \sqrt{\frac{12\mu x}{\rho U}}$$

The local skin friction coefficient is defined as

$$c_f = \frac{\tau_w[x]}{\tfrac{1}{2}\rho U^2} = \frac{\mu}{\tfrac{1}{2}\rho U \delta[x]} = \frac{\mu}{\tfrac{1}{2}\rho U}\sqrt{\frac{\rho U}{12\mu x}} = \sqrt{\frac{\mu}{3\rho U x}}$$

The friction drag coefficient for a flat plate of length ℓ and width b is

$$C_D = \frac{D}{\tfrac{1}{2}\rho U^2 A} = \frac{b\int_0^{\ell} \tau_w[x]\,\mathrm{d}x}{\tfrac{1}{2}\rho U^2 b\ell} = \frac{1}{\ell}\int_0^{\ell} c_f\ \mathrm{d}x = \frac{1}{\ell}\int_0^{\ell}\sqrt{\frac{\mu}{3\rho U x}}\ \mathrm{d}x = \frac{1}{\ell}\sqrt{\frac{\mu}{3\rho U}}\,2\sqrt{\ell} =$$

$$\sqrt{\frac{4\mu}{3\rho U\ell}} = \sqrt{\frac{4}{3Re_l}}$$

QUESTION 2

If the boundary layer on the hood of your car behaves as one on a flat plate, estimate how far from the front edge of the hood the boundary becomes turbulent if the transition point occurs at $\mathrm{Re} \approx 5 \times 10^5$. How thick is the boundary layer at this location if the boundary layer thickness is given by $\delta = 5\sqrt{\frac{x\mu}{\rho U}}$? If the momentum thickness is given by $\frac{\Theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$, use the momentum integral equation for flow over a flat plate ($\tau_w = \rho U^2 \frac{d\Theta}{dx}$) to obtain the drag force per unit width.

The Reynolds number for a flat plate is defined as $\mathrm{Re} = \rho U x / \mu$. The transition occurs at $\mathrm{Re} = 500000$.

Hence $x = \frac{\mathrm{Re}\ \mu}{\rho U} = \frac{500000 * 1.79 * 10^{-5}}{1.225\ U} = \frac{7.3}{U}$

The boundary layer thickness for the Blasius's case is given by

$$\delta = 5\sqrt{\frac{x\mu}{\rho U}} = 5\sqrt{\frac{\frac{7.3}{U}\mu}{\rho U}} = 5\sqrt{\frac{7.3 * 1.79 * 10^{-5}}{1.225 * U^2}} = \frac{0.052}{U}$$

QUESTION 3

To obtain an expression of the boundary layer thickness of a turbulent boundary layer it is suggested to use the Blasius's formula for the friction factor of a smooth-pipe:

$$f = 0.079\,\mathrm{Re}^{-1/4},$$

where the friction factor is defined as $f = \dfrac{\tau}{\frac{1}{2}\rho \bar{u}^2}$**, and the Reynolds number** Re **is defined with respect to the radius of the pipe. If the radius of the pipe is assumed equivalent to the boundary layer thickness** δ **on a flat plate, obtain an expression for** δ**. Obtain an expression for the total drag force** F_D **on one side of a plate of length** ℓ **and unit width.**

For a pipe of radius R

$$\tau_w = \frac{1}{2}\rho\, \bar{u}^2 0.079 \left(\frac{\mu}{\bar{u}\; 2R\rho}\right)^{(1/4)} = \text{const}\; \rho\; u_{\max}^2 \left(\frac{\mu}{\rho\; u_{\max} R}\right)^{(1/4)}$$

since for a particular velocity profile $u_{\max}$ is proportional to $\bar{u}$.
If R is now assumed equivalent to the boundary layer thickness on the flat plate, δ, we have

$$\boxed{\tau_w = \text{const}\; \rho\; u_{\mathrm{m}}^2 \left(\frac{\nu}{u_{\mathrm{m}}\delta}\right)^{(1/4)}} \quad \text{where } \nu = \frac{\mu}{\rho}.$$

Before we employ the integral momentum equation $\boxed{\tau_w = \rho U^2 \dfrac{\mathrm{d}\Theta}{\mathrm{d}x}}$

consider $\Theta = \displaystyle\int_0^{\delta(x)} \frac{u}{U}\left(1 - \frac{u}{U}\right)\mathrm{d}y = \delta(x)\int_0^1 \frac{u(\eta)}{U}\left(1 - \frac{u(\eta)}{U}\right)\mathrm{d}\eta$

where the dimensionless coordinate $\eta = \dfrac{y}{\delta}$. Hence $\boxed{\Theta = \delta\; \text{const}_1}$.

Combining the equations in the boxes we obtain:

$$\text{const}\; \rho\; u_{\mathrm{m}}^2 \left(\frac{\nu}{u_{\mathrm{m}}\delta}\right)^{(1/4)} = \rho U^2 \text{const}_1 \frac{\mathrm{d}\delta}{\mathrm{d}x}$$

which simplifies to $\delta^{1/4}\dfrac{\mathrm{d}\delta}{\mathrm{d}x} = \text{const}_2\left(\dfrac{\nu}{u_{\mathrm{m}}}\right)^{(1/4)}$. Integrating with respect to x yields

$\dfrac{4}{5}\delta^{5/4} = \text{const}_2\left(\dfrac{\nu}{u_{\mathrm{m}}}\right)^{(1/4)} x + C.$ If we neglect the constant C, we obtain

$$\delta = \text{const}\left(\frac{\nu}{u_{\mathrm{m}}}\right)^{1/4} x^{4/5}.$$

QUESTION 4

Consider an airfoil with chord length of $1.5\,\mathrm{m}$. For the case where the $\mathrm{Re}_c = 3.1 \times 10^6$, estimate: (a) the laminar boundary layer thickness at the trailing edge and (b) the net laminar skin-friction drag coefficient for the airfoil.

The boundary-layer thickness for incompressible laminar flow over a flat plate at zero angle of attack is

$$\delta = \frac{5.0\,x}{\sqrt{\mathrm{Re}_x}}$$

Applying above equation at the trailing edge, where $x = c,$ we hav

$$\delta = \frac{5.0\,c}{\sqrt{\mathrm{Re}_c}} = \frac{5.0 \times 1.5}{\sqrt{3.1 \times 10^6}} = 0.00426\text{ m}$$

Notice, how thin the boundary layer is; at the traling edge, where its thickness is the largest, the boundary layer is only 4.3 mm.

The local skin friction coefficient is $c_f = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$.

Letting C_f denote the skin friction drag coefficient, we obtain

$$C_f = \frac{1}{c}\int_0^c c_f\,dx = \frac{1}{c}(0.664)\sqrt{\frac{\mu}{\rho V}}\int_0^c \sqrt{x}\,dx = \frac{1.328}{c}\sqrt{\frac{\mu c}{\rho V}}$$

$$\Rightarrow C_f = \frac{1.328}{\sqrt{\mathrm{Re}_c}}$$

Hence, the skin friction drag coefficient is $C_f = \dfrac{1.328}{\sqrt{3.1 \times 10^6}} = 7.54 \times 10^{-4}$

This is for a single surface. Taking both surfaces into account $C_f = 0.0015$

Do you think it was reasonable to assume a laminar boundary layer? Estimate the boundary layer thickness and the skin-friction drag for the case of a turbulent boundary layer.

For the relatively high Reynolds number of 3.1×10^6, the boundary layer over the airfoil will be turbulent, not laminar. So we expect that the experimental skin-friction drag coefficient to be higher.

We replace the airfoil with a flat plate at zero angle of attack. The boundary-layer thickness at the trailing edge, where $x = c$ and $\mathrm{Re}_x = \mathrm{Re}_c = 3.1\times10^6$ is

$$\delta = \frac{0.37\,x}{\left(\mathrm{Re}_x\right)^{1/5}} = \frac{0.37\times1.5}{\left(3.1\times10^6\right)^{1/5}} = 0.0279 \text{ m}$$

The turbulent boundary layer is 2.79 cm at the trailing edge, and it is much thicker than the laminar boundary layer thickness.

The skin-friction coefficient is given by

$$C_f = \frac{0.074}{\left(\mathrm{Re}_c\right)^{1/5}} = \frac{0.074}{\left(3.1\times10^6\right)^{1/5}} = 0.00372$$

This is for a single surface. Taking both surfaces into account $C_f = 0.00744$
This result is a factor of five larger than for the laminar boundary layer. It demonstrates the considerable increase in skin friction caused by the a turbulent boundary layer in comparison to that caused by a laminar boundary layer.

What is the effect of the turbulent boundary layer on the drag force?
For streamlined bodies, the drag coefficient increases when the boundary layer becomes turbulent because most of the drag is due to the shear force, which is greater for turbulent flow than for laminar flow. On the other hand, the drag coefficient for a relatively blunt object, such as a cylinder or sphere, actually decreases when the boundary layer becomes turbulent. A turbulent boundary layer can travel further along the surface into the adverse pressure gradient on the rear portion of the cylinder before separation occurs. This is because the location of separation, the width of the wake region behind the object, and the pressure distribution on the surface depend on the nature of the boundary layer flow. Compared with a laminar boundary layer, a turbulent boundary layer flow has more kinetic energy and momentum associated with it because: (1) the velocity profile is fuller, more nearly like the ideal uniform profile, and (2) there can be considerable energy associated with the swirling, random components of the velocity. The result is a thinner wake and smaller pressure drag for turbulent boundary layer flow.