FREDERICK UNIVERSITY CYPRUS

Department of Mechanical Engineering

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Homework Assignment #5

QUESTION 1

The static pressure to stagnation pressure ratio at a point in a flow stream is measured with a Pitot-static tube (see Fig. 3.6) as being equal to 0.82. The stagnation temperature of the fluid is 68 °F. Determine the flow velocity if the fluid is (a) air, (b) helium.

SOLUTION

We consider both air and helium, flowing as described above, to act as ideal gases with constant specific heats. Then, we can use any of the ideal gas relationships developed in this chapter. To determine the flow velocity, we can combine Eqs. 11.36 and 11.46 to obtain

$$V = \text{Ma}\sqrt{RTk} \tag{1}$$

By knowing the value of static to stagnation pressure ratio, p/p_0, and the specific heat ratio we can obtain the corresponding Mach number from Eq. 11.59, or for air, from Fig. D.1. Fig. D.1 cannot be used for helium, since k for helium is 1.66 and Fig. D.1 is for $k = 1.4$ only. With Mach number, specific heat ratio, and stagnation temperature known, the value of static temperature can be subsequently ascertained from Eq. 11.56 (or Fig. D.1 for air).

(a) For air, $p/p_0 = 0.82$; thus from Fig. D.1,

$$\text{Ma} = 0.54 \tag{2}$$

and

$$\frac{T}{T_0} = 0.94 \tag{3}$$

Then, from Eq. 3

$$T = (0.94)[(68 + 460)\ {}^\circ\text{R}] = 496\ {}^\circ\text{R} \tag{4}$$

and using Eqs. 1, 2, and 4 we get

$$V = (0.54)\sqrt{[1.716 \times 10^3\ (\text{ft}\cdot\text{lb})/(\text{slug}\cdot{}^\circ\text{R})](496\ {}^\circ\text{R})(1.4)[1\ (\text{slug}\cdot\text{ft})/(\text{lb}\cdot\text{s}^2)]}$$

or

$$V = 590\ \text{ft/s} \qquad \textbf{(Ans)}$$

(b) For helium, $p/p_0 = 0.82$ and $k = 1.66$. By substituting these values into Eq. 11.59 we get

$$0.82 = \left\{\frac{1}{1 + [(1.66 - 1)/2]\,\text{Ma}^2}\right\}^{1.66/(1.66-1)}$$

or

$$\text{Ma} = 0.499$$

From Eq. 11.56 we obtain

$$\frac{T}{T_0} = \frac{1}{1 + [(k - 1)/2]\text{Ma}^2}$$

Thus,

$$T = \left\{\frac{1}{1 + [(1.66 - 1)/2](0.499)^2}\right\}[(68 + 460)\ {}^\circ\text{R}] = 488\ {}^\circ\text{R}$$

From Eq. 1 we obtain

$$V = (0.499)\sqrt{[1.242 \times 10^4(\text{ft}\cdot\text{lb})/(\text{slug}\cdot{}^\circ\text{R})](488\ {}^\circ\text{R})(1.66)[1(\text{slug}\cdot\text{ft})/(\text{lb}\cdot\text{s}^2)]}$$

or

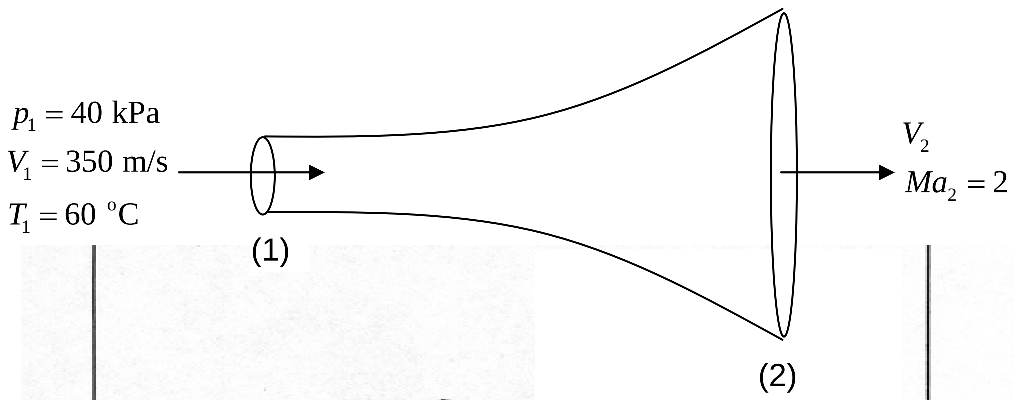
$$V = 1580 \text{ ft/s} \qquad \textbf{(Ans)}$$

Note that the isentropic flow equations and Fig. D.1 for $k = 1.4$ were used presently to describe fluid particle isentropic flow along a pathline in a stagnation process. Even though these equations and graph were developed for one-dimensional duct flows, they can be used for frictionless, adiabatic pathline flows also.

Furthermore, while the Mach numbers calculated above are of similar size for the air and helium flows, the flow speed is much larger for helium than for air because the speed of sound in helium is much larger than it is in air.

QUESTION 2

At section (1), in the isentropic flow of carbon dioxide, $p_1 = 40$ kPa (abs), $T_1 = 60$ °C, and $V_1 = 350$ m/s**. Determine the flow velocity** V_2, in m/s**, at another section (2), where the Mach number is 2.0. Also calculate the section area ratio,** A_2/A_1**.**



$$V_2 = Ma_2\, c_2 = 2\, c_2 = 2\sqrt{kRT_2} \quad (1)$$

where for CO_2, $k = 1.30$ and $R = 188.9 \frac{N\cdot m}{kg\cdot K}$ (see Table 1.8)

Also, from Eq. 11.56

$$\frac{T_2}{T_{0,2}} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2} \text{ and } \frac{T_1}{T_{0,1}} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}$$

, where for isentropic flow, $T_{0,1} = T_{0,2}$. Thus, by dividing these equations we obtain

$$\frac{T_2}{T_1} = \frac{1+\left(\frac{k-1}{2}\right)Ma_1^2}{1+\left(\frac{k-1}{2}\right)Ma_2^2} \quad (2)$$

However,

$$Ma_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{350\ \frac{m}{s}}{\sqrt{188.9\frac{N\cdot m}{kg\cdot K}(1.3)(273+60)K}} = 1.22$$

Thus, from Eq. (2),

$$T_2 = (273+60)K\, \frac{\left[1+\left(\frac{1.3-1}{2}\right)(1.22)^2\right]}{\left[1+\left(\frac{1.3-1}{2}\right)(2)^2\right]} = 255\text{ K, so from Eq. (1)}$$

$$V_2 = 2\left[1.3\left(188.9\frac{N\cdot m}{kg\cdot K}\right)(255)\right]^{1/2} = \underline{\underline{500\ \frac{m}{s}}}$$

Also, (see Eq. 11.71)

$$\frac{A_2}{A_1} = \frac{\left(\frac{A_2}{A^*}\right)}{\left(\frac{A_1}{A^*}\right)} = \frac{\frac{1}{Ma_2}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_2^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}}{\frac{1}{Ma_1}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_1^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}} = \frac{Ma_1}{Ma_2}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_2^2}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{k+1}{2(k-1)}}$$

so that with

$$\frac{k-1}{2} = \frac{1.3-1}{2} = 0.15,\quad \frac{k+1}{2(k-1)} = \frac{1.3+1}{2(1.3-1)} = 3.83,\quad Ma_1 = 1.22, \text{ and } Ma_2 = 2$$

we obtain

$$\frac{A_2}{A_1} = \frac{1.22}{2}\left[\frac{1+0.15(2)^2}{1+0.15(1.22)^2}\right]^{3.83} = \underline{\underline{1.71}}$$

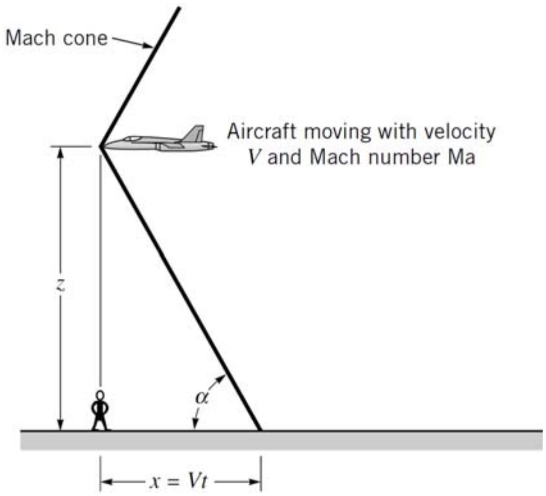
QUESTION 3

Using the compressible Bernoulli's equation and mass conservation find an expression for the speed of sound assuming isentropic flow of an ideal gas (use the relations $p\rho^{-\gamma} = const$ and $p = \rho RT$). Hence estimate the speed of sound in air at ambient conditions. ($R = 286.9 J/kg/K, \gamma = 1.401$)

QUESTION 4

a. **Consider a weak pressure pulse travelling through a fluid at rest. Determine the speed of the pulse using a control volume analysis. Hence, determine the speed of sound in an ideal gas.**

b. **An aircraft is cruising at 1000-m elevation z above you moves past in a flyby. How many seconds after the plane passes overhead do you expect to wait before you hear the aircraft if it is moving with a Mach number equal to 1.5 and the ambient temperature is $20\ ^\circ\text{C}$ (for air $R = 286.9\ \text{J/(kg}\cdot\text{K)},\ \text{k=1.401}$).**



the cone reaches the observer, the "sound" of the aircraft is perceived. The angle α in Fig. E11.4 is related to the elevation of the plane, z, and the ground distance, x, by

$$\alpha = \tan^{-1}\frac{z}{x} = \tan^{-1}\frac{1000}{Vt} \qquad \textbf{(1)}$$

Also, assuming negligible change of Mach number with elevation, we can use Eq. 11.39 to relate Mach number to the angle α. Thus,

$$\text{Ma} = \frac{1}{\sin\alpha} \qquad \textbf{(2)}$$

Combining Eqs. 1 and 2 we obtain

$$\text{Ma} = \frac{1}{\sin\left[\tan^{-1}(1000/Vt)\right]} \qquad \textbf{(3)}$$

The speed of the aircraft can be related to the Mach number with

$$V = (\text{Ma})c \qquad \textbf{(4)}$$

where c is the speed of sound. From **Table B.4**, $c = 343.3$ m/s. Using Ma = 1.5, we get from Eqs. 3 and 4

$$1.5 = \frac{1}{\sin\left\{\tan^{-1}\left[\frac{1000\text{ m}}{(1.5)(343.3\text{ m/s})t}\right]\right\}}$$

or

$$t = 2.17\text{ s} \qquad \textbf{(Ans)}$$