Analytical Model for Geometrical Characteristics Control of Laser Sintered Surfaces

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Selective laser sintering (SLS) is an additive rapid manufacturing technique where a high-power laser is used to fuse micro- and nano-particles into a specified 3-dimensional geometry. The goal of this work was to develop an analytical model for the SLS manufacturing process in order to control the geometrical characteristics of the sintered areas when iron/copper (Fe/Cu) powder alloy is used on a flat substrate. Powder particles are subject to melting by the laser energy and form a liquid globule that solidifies as the laser beam spot moves on the substrate. The model is developed by considering lumped mass and energy balances and fluid dynamic equilibrium of the sintered material. It is assumed that the process is two-dimensional and axisymmetric. It is further assumed that the energy delivered by the laser is used to sinter the material rising its temperature up the melting point, while the energy lost due to conduction in the metal substrate is very small. The sintered area geometry is parameterized and the parameters are obtained by solving a system of nonlinear algebraic equations.

I. INTRODUCTION

Selective laser sintering (SLS) offers unique advantages over conventional thermo-mechanical processes, and for this reason it is one of the leading commercial rapid prototyping processes used so far. The process involves formation of fabricating solid objects by selectively fusing powder of successive layers according to numerically defined cross-sectional geometry. Some successful results have been obtained with different powder mixtures, such as Fe-Cu, WC-Co, TiC-Ni/Co/Mo, TiCN-Ni, TiB_2-Ni, ZrB_2-Cu and Fe_3C-Fe, each containing two metal powders and fusing only the powder having the lower melting point [1], [2]. Several physical phenomena are involved in the SLS process. Heat generation and transfer (the heating of the powder bed and the cooling of the sintered sample); microstructure evolution (porosity evolution and phase changes); fluid effects (molten binder flowing in the solid consolidate) and mechanical issues (non-uniform distribution of thermal strains during the cooling stage may cause residual stresses and distortions of parts produced) [1].

In these coexisting physical phenomena, the thermal problem is dominant [3]. The other problems, caused directly or indirectly by this thermal process, occur at different processing stages. It is clear from the above that knowing the temperature distribution and evolution is essential to understand the SLS process.

Sintering takes place if the powder bed is irradiated by the moving laser source up to the temperature at which the binder phase melts. After the laser has moved away, the sintered sample cools down. The thermal process in SLS may be summarized in four main stages: Energy input and absorption, heating of the powder bed, binder melting and sintering, and cooling of the sintered sample.

Fe–Cu is a well-described material system for liquid-phase sintering (LPS) because this material system has good wetting. Because the reflectivity of Cu is higher than that of Fe, the SLS of Fe-Cu requires careful control of grain size and mixing ratio of the powders to avoid melting of Fe before Cu. It is an ideal metallurgy material utilized for fabricating tooling because of its good mechanical properties: the hard Fe lattice is dispersed in a ductile Cu matrix. Moreover, the price of these two powders is relatively low.

II. MATHEMATIC MODELING

A. *Experimental procedure*

SLS Process and Physical Description: The material to be sintered consists of a mixture of two powders: a high-melting-point powder, called the structural powder, and a powder having a lower melting point, called the binder (Cu). This mixture is spread as a thin layer, normally 0.1–0.5 mm in thickness. The mixed powders are then heated by a moving laser beam to the temperature at which the binder melts. Another important characteristic is that the sintering time is extremely short in SLS, because the moving laser beam supplies energy to each cross-sectional location only for about 1 ms–0.1 s. Once the binder is molten and has flow into the pores between the unmolten structural particles, the system cools down. No further densification can take place in such a short time interval.

Localized heating of small volumes of powder during SLS is shown schematically in Figure 1. The five independent process parameters that are most influential in governing the intensity and method of the energy delivered to the powder surface are: laser power $\dot{Q}_{in}$, laser beam distribution spot size SP, laser velocity v, hatch spacing HS and scan line length L. Phenomena such as wet-ability, viscosity and flow play an important role to the metallurgical process. In the nomenclature all the parameters of the model are listed.

B. *Theoretical model*

As mentioned, in the previous section, the sintering process is achieved by a moving laser beam, moving along the span of the powder-bed with velocity v, delivering an amount of energy to the powder-bed. The energy is conducted first through the powder, which is assumed to be a continuum medium, and then through the metal-substrate. In view of the small distribution spot size SP of the laser beam, the energy delivery process is modeled as a point source delivering a constant power $\dot{Q}_{in}$. In view of the rapid movement of the point source, we can further assume that the power of the laser beam is distributed simultaneously and evenly along the span of the surface of the powder-bed for a time period $\tau = L/v$, where L is the span of the powder-bed (Figure 1), delivering a total energy of $Q_{in} = \dot{Q}_{in}\,\tau$ (Figure 1). The net energy delivered per unit length is defined as $q_{in} = Q_{in}/L = \dot{Q}/v$. As the heat is conducted through, a phase-change takes place. In particular, an interface is developed with temperature T_{melt} that separates the sintered material (liquid) from the solid material (powder and metal).

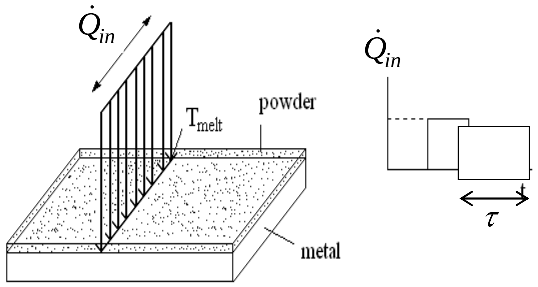

Figure.1: Interaction of laser irradiation with metal powder. A laser beam with power $\dot{Q}_{in}$ delivers an amount of energy $Q_{in} = \dot{Q}_{in}\tau$ to the powder layer.

The interface travels through the powder-layer and metal-substrate and its path determines the final profile of the sintered material. The objective of this work is to determine this profile. The problem is considered to be a two-phase problem. Taking a cross section of the powder-metal substrate bed, first phase begins when the heat source (laser beam) 'hits' the powder layer. Immediately after the forming of the melted powder interface takes place traveling through the powder layer, when it reaches the metal substrate the second phase of the model begins. By applying mass and energy conservation, and assuming that the contact angles of the three-phase contact lines associated with the two interfaces are known, we develop a system of algebraic equations through which the parameters of the profile can be obtained. The symbols used are all listed in Table I.

B1. Initial phase.

The model is developed by considering mass conservation, energy balance and fluid dynamic equilibrium of the sintered material on each phase. The interface between the gas/sintered-material is assumed to take the shape of circular arc.

$$\gamma_{gl} = \gamma_{lp}\cos(\pi - \vartheta_p) + \gamma_{gl}\cos(\pi - \vartheta_p - \vartheta_l) \quad (1)$$

The mass of the powder sintered which is associated with the area A′ must be equal to the mass of the molten metal occupying the area (A′-A).

$$\rho_p A' L = \rho_m (A' - A) L \quad (2)$$

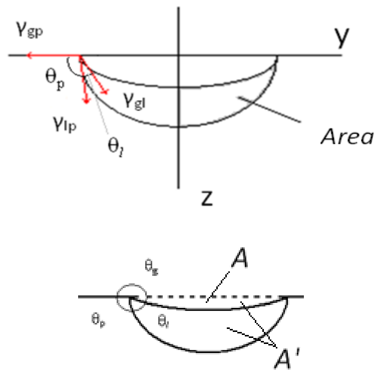

Figure.2: Miniscus forming as the melted material interface traveling through the powder layer.

Energy absorbed for sintering has two components. The energy required in order to increase the internal energy of the material to T_{melt} and the latent heat required to melt it. By applying the energy conservation on the 2-dimensional profile we have.

$$q_{in} = q_p + q_{melt} \quad (3)$$

$$q_{melt} = p_m \Delta H (A' - A) + \rho_m C_p A' (T_{melt} - T_o)$$

Where energy transferred due to heat conduction in the powder 1-d transient heat conduction.

$$q_p = \frac{K_p \Delta T}{(\pi a)^{1/2}} c\sqrt{t}$$

For the initial phase the geometrical relation required for the model are:

$$A = r_p^2 \cos^{-1}\frac{r_p + h_p}{r_p} - (r_p + h_p)\sqrt{2 r_p h_p - h_p^2}$$

$$\vartheta_g + \vartheta_l + \vartheta_p = 2\pi$$

B2. Equilibrium shape of the final profile

In order to determine the final profile of the sintered material, we assume a shape for each segment of the profile. In particular: a. in view of the small thickness of the powder-layer the side segments, which are interfaces between the powder and the melted material, are assumed to be straight (Figure 3) b. the top and bottom segments, which are interfaces between the gas/sintered-material and sintered-material/metal-substrate (Figure 3) respectively, are assumed to take the shape of circular arcs of circles.

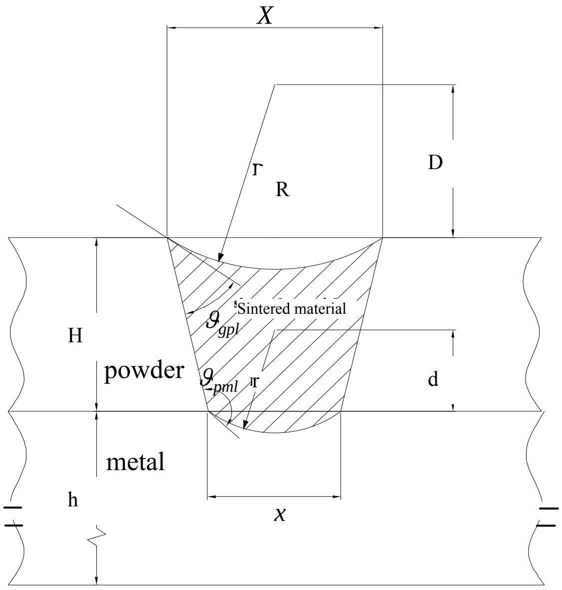

Figure 3 : Geometrical parameters of the final profile.

This is justified in view of the fact that a circular arc is an asymptotic solution to the Laplace-Young equation. The Laplace-Young equation describes the shape of a curved interface with uniform tension γ separating two stationary fluids. The boundary conditions of the Laplace-Young equation are dictated from the contact angles of the trio melted-material/powder/air and solid/powder/melted-material systems. These depend only on the materials and in no other way on the conditions of the problem [4]. The contact angles can be deduced from an independent experiment using equilibrium on y-axis for the contact lines along the three media (Figure 4):

$$\gamma_{gp} = \gamma_{gl} \cos\varphi + \gamma_{lp} \cos\vartheta_{gpl}$$

$$\gamma_{sl} + \gamma_{pl} \cos\left(\pi - \left(\vartheta_{pml} - \varphi'\right)\right) = \gamma_{ls} \cos\varphi'$$

Based on the geometrical formulation of the model we deduce the followings trigonometric relations between the contact angles and the geometrical parameters (Figure 3):

$$\arctan\left(\frac{H}{(X-x)/2}\right) - \left(\frac{\pi}{2} - \arctan\frac{D}{X/2}\right) = \vartheta_{gpl} \quad (4)$$

$$\frac{\pi}{2} + \left(\pi - \arctan\left(\frac{H}{(X-x)/2}\right) - \arctan\frac{d}{x/2}\right) = \vartheta_{pml} \quad (5)$$

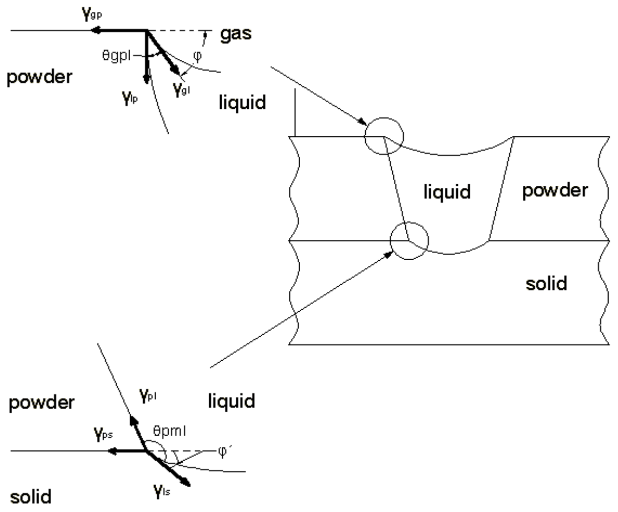

Figure 4: Equilibrium at the line of contact of three different media.

In what follows, we will also require expressions for the areas associated with each section of the profile. These sections are clearly indicated in Figure 5, and the expressions can be found in [5]:

$$A = R^2 \cos^{-1}\left(\frac{D}{R}\right) - \frac{XD}{2} \quad (6)$$

$$A' = \frac{(X+x)}{2} H - A \quad (7)$$

$$A_m = r^2 \cos^{-1}\left(\frac{d}{r}\right) - \frac{xd}{2} \quad (8)$$

where

$$R = \sqrt{D^2 + (X/2)^2} \text{ and } r = \sqrt{d^2 + (x/2)^2} \quad (9)$$

In order to obtain the final profile, it is sufficient to determine the parameters D, d, X and x. Equations (4) and (5) along with the mass and energy conservation equations, which will be developed in the next two sections, constitute a system of equations for the four unknown parameters.

B2. Mass Balance

Mass conservation of the molten material suggests that the mass of the powder sintered, which is associated with the trapezoid with area $(A + A')$, must be equal to the mass of the molten metal occupying the area A' :

$$\rho_m A' L = \rho_p (A + A') L \quad (10)$$

Here, we have assumed that the density of the molten material is equal to the density of the metal. The areas A and A' are shown clearly in Figure 5 and the parameters involved are identified in the previous section.

B3. Energy delivery process

Many studies have been reported on laser interaction with solids, such as laser cutting and laser welding [2]. In SLS process a linearly traversing laser beam selectively fuse powder particles. Applying the energy conservation [6] per unit length on the 2-dimensional profile (Figure 5), we obtain the following equation:

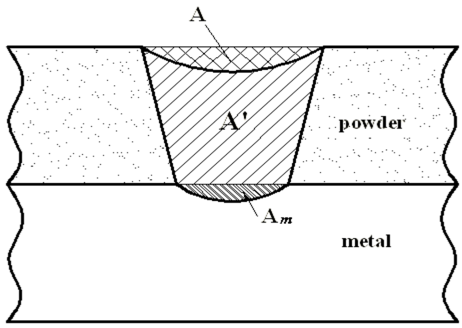

Figure 5: Geometry areas A and A΄ used in the mass balance.

$$q_{in} = q_p + q_m + q_{melt} \quad (11)$$

where q_{in} is the energy input from the laser, q_p and q_m denote the energy transferred due to heat conduction in the powder and the metal-substrate respectively, and q_{melt} is the energy stored due to the sintering of the material.

$$q_p = \frac{k_p \Delta T}{\sqrt{\pi \alpha_p}} 2 l_p \sqrt{t}$$

$$q_m = \frac{K_m \Delta T}{\sqrt{\pi \alpha_m}} c_m \sqrt{t}$$

The energy absorbed for sintering (q_{melt}) has two components. The energy required in order to increase the internal energy of the material to T_{melt} and the latent heat required to melt it:

$$q_{melt} = \rho_m (A' + A_m) \Delta H + \rho_m A' C_p (T_{melt} - T_o) + \rho_m A_m C_m (T_{melt} - T_o) \quad (9)$$

Considering the instant fusion of the powder, the relatively fast solidification of the molten material and the small duration of the energy delivery process τ, we can assume that the energy due to conduction can be neglected. This can be further justified in view of the fact that: a. during the short period of time $\tau = 1 - 100$ ms, the heat transferred due to conduction would be very small b. the thermal conductivity of the powder k_p is very small c. the latent heat ΔH of the material under consideration is large, hence most of the energy delivered by the laser is contained in q_{melt}.

C. Numerical results

The system of equations (4), (5), (7) and (10) along with (6), (7), (8), (9), and (9) is a consistent system of equations for the four unknowns D, d, X and x. The system of equations is solved numerically using the *fsolve* function of the commercial software Matlab. The material and experimental properties used have been compiled from the literature [1, 7]:

L	0.3 m
H	0.0003 m
w	0.3 m
h	0.01 m
C_p	480 J/K Kg
C_m	450 J/K Kg
ΔH	213000 J/Kg
q_{in}	3000 J/m
τ	0.1 s
ϑ_{gpl}	48 degrees
ϑ_{pml}	136.5 degrees
ρ_p	7000 kg/ m^3
ρ_m	6500 kg/ m^3

Table 1 Sintering parameters applied in the numerical simulation.

The results obtain are as follows: X=0.0005364 m, x=0.0005165 m, D=0.0003184 m, d=0.0002618 m which correspond to R=0.00042 m and r=0.00037 m. The results are in qualitative agreement with experimental results.

III. CONCLUSIONS

In this study, a 2-D analytical geometry model for the SLS manufacturing processes is proposed in order to obtain the geometrical characteristics of the sintered area when iron/copper (Fe/Cu) powder alloy is processed on a flat substrate. The energy delivery process is modeled as a line source delivering a constant power. The objective of the work is to obtain the final profile of the sintered material. The final profile is parameterized by assuming that the side segments are straight, which is justified in view of the small thickness of the powder-layer. Furthermore, the top and bottom interfaces, which separate the gas/sintered-material and sintered-material/metal-substrate respectively, are assumed to take the shape of circular arcs of circles. We also assume that the energy transfer due to conduction can be neglected in view of the instant fusion of the powder and the relatively fast solidification of the molten material. By applying mass and energy conservation, and assuming that the contact angles of the three-phase contact lines associated with the two interfaces are known, a system of four nonlinear algebraic equations is obtained through which the parameters of the profile can be obtained. The system of the four nonlinear algebraic equations is solved numerically using material and experimental properties from the literature. The results obtained are in good agreement with experimental measurements of the shape of the sintering zone.

IV. ACKNOWLEDGEMENTS

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Nomenclature

C_p	thermal capacitance of powder	J/kgK
C_m	thermal capacitance of metal	J/kgK
D, d	center to chord distance	m
ΔH	latent heat	J/Kg
HS	hatch spacing	m
H	metal substrate height	m
h	powder height	m
k_p	thermal conductivity of the powder	W/m * K
k_m	thermal conductivity of metal	W/m * K
L	scan line length	m
$\dot{Q}_{in}$	laser power	W (watt)
q_{in}	energy delivered by laser	J/m
q_p	energy due to conduction in the powder per unit length.	J/m
q_m	energy due to conduction in the metal substrate	J/m
R, r	radius of circle	m
S	dimensionless shape factor	
SP	laser spot size	m/s
T_o	ambient temperature	K
T_{melt}	powder melting temperature	K
v	laser velocity	m/s
w	width of the metal substrate	m
X , x	chord of circle	m
c	circumference	m
t	time	s

Greek Symbols

α	thermal diffusivity	m²/s
γ	surface tensions	N/m
τ	time period	s
ρ_p	powder density	Kg/m^3
ρ_m	metal density	Kg/m^3

Subscripts

g	gas
p	powder
l	liquid
m	metal

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