FREDERICK UNIVERSITY CYPRUS

AMAT 223 FINAL EXAM PRACTICE PROBLEMS SPRING 2009

QUESTION 1

Let $\vec{u} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{v} = \vec{j} + \vec{k}$.

a) Find $|\vec{u}|, |\vec{v}|$
b) Find a unit vector parallel to $\vec{u}$.
c) Find $\vec{u} \bullet \vec{v}$ and $\vec{u} \times \vec{v}$.
d) Are the two vectors perpendicular to each other? Are they parallel? Why or why not?
e) Find the equation of the plane that is perpendicular to $\vec{u}$ and passes through the point $P(1,0,1)$.
f) Find a unit vector that is perpendicular to both $\vec{u}$ and $\vec{v}$.

QUESTION 2

Consider the points $A(-1,1,2)$, $B(1,0,2)$, and $C(-1,2,-1)$

a) Let $\vec{w} = \vec{AB}$ and $\vec{s} = \vec{AC}$. Find $\vec{w}, \vec{s}$ and $|\vec{w}|$.
b) Find a vector perpendicular to both $\vec{w}$ and $\vec{s}$.
c) Write down the equation of the plane containing the points A, B and C.

QUESTION 3

a) Consider the surface with equation $2x - 3y + 5z = 12$:
i) Name the surface.
ii) Find and name its trace on the xy-plane. Sketch this trace on the xy-plane.
iii) Find and name its trace on the yz-plane. Sketch this trace on the yz-plane.

b) Consider the surface with equation $z = \frac{x^2}{9} + y^2$.
i) Find and name its trace on the plane $x = 3$. Sketch the vertical projection of this trace on the yz-plane.
ii) Find and name its trace on the plane $z = 1$. Sketch the vertical projection of this trace xy-plane.
iii) Find and name its trace on the plane $z = -1$. Sketch the vertical projection of this trace on the xy-plane.

c) Consider the surface with equation $x^2 + y^2 = 1$:
i) Name the surface
ii) Find and name its trace in the plane $z = 5$. Sketch the vertical projection of this trace on the xy-plane.
iii) Find and name its trace in the plane $z = -2$. Sketch the vertical projection of this trace on the xy-plane.

d) Consider the surface with equation $z = \ln x$:

iv) Name the surface

v) Find its trace in the plane $y = 5$. Sketch the vertical projection of this trace on the xz-plane.

vi) Find its trace in the plane $y = -2$. Sketch the vertical projection of this trace on the xz-plane.

QUESTION 4

Consider the function $F(x, y, z) = y^3 + 5xy - 4xz + z^2$. The function $z = f(x, y)$ is defined implicitly by the equation $F(x, y, z) = 2$, i.e. by the equation $y^3 + 5xy - 4xz + z^2 = 2$.

a) Find $F_{x,}\ F_y, F_z$, and $\vec{\nabla} F$.

b) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

c) Consider the surface defined (implicitly) by the above equation, $F(x, y, z) = y^3 + 5xy - 4xz + z^2 = 2$. Find the equation of the tangent plane to this surface at the point $P(1,0,1)$.

QUESTION 5

I. Consider the function $f(x, y) = x^3 - 3x + y^3 - 3y$.

a) Find $f_x, f_y, f_{xy}, f_{xx}, f_{yy}$ and D.

b) Find and classify the critical points of the above function $f(x, y)$.

II. Consider the function $f(x, y) = 3y^2 + x^2 - 2xy - 4y$.

c) Find $f_x, f_y, f_{xy}, f_{xx}, f_{yy}$ and D.

d) Find and classify the critical points of the above function $f(x, y)$.

III. Use the method of Lagrange Multipliers to find the maximum and /or minimum value(s) of the function $f(x, y) = -x^2 + y^2$ subject to the constraint $2x^2 + 2y^2 = 8$.

IV. Use the method of Lagrange Multipliers to find the maximum and /or minimum value(s) of the function $f(x, y) = 4x + 3y$ subject to the constraint $2x^2 + 3y^2 = 11$.

QUESTION 6

Consider the following vector fields:

$$\vec{F}(x,y) = (2x-5y)\ \hat{i}\ +\ (-5x+3y^2)\ \hat{j}$$

$$\vec{H}(x,y) = (5x-2)\ \hat{i}\ +\ (3x^2+6y)\ \hat{j}$$

a) Find $\vec{\nabla}\times\vec{F}$ and $\vec{\nabla}\bullet\vec{H}$

b) Is the given vector field $\vec{F}(x,y)$ conservative? If it is, find an $f(x,y)$ such that $\vec{F} = \vec{\nabla}f$.

c) Is the given vector field $\vec{H}(x,y)$ conservative? If it is, find a $h(x,y)$ such that $\vec{H} = \vec{\nabla}h$.

d) Let C_1 be the curve $y = x^3$ from the point $(0,0)$ to the point $(1,1)$. Evaluate the line integral $\int_{C_1}(5x-2)\ dx\ +\ (3x^2+6y)\ dy$.

e) Let C_2 be the curve $y = x^2+3x+1$ from the point $(0,1)$ to the point $(1,5)$. Evaluate the line integral $\int_{C_2}(2x-5y)\,dx+(-5x+3y^2)\,dy$.

QUESTION 7

a) Let $f(x,y)$ be a function for which the following double integrals exist.

Rewrite each double integral with the order of integration reversed:

(i) $\int_8^{45}\int_4^{23} f(x,y)\ dy\ \ dx$ (ii) $\int_0^4\int_{y/4}^1 f(x,y)\ dx\ dy$

(iii) $\int_1^3\int_0^{\ln x} f(x,y)\ dy\ dx$ (iv) $\int_0^1\int_0^{\sqrt{y}} f(x,y)\ dx\ dy$

(v) $\int_0^1\int_y^1 f(x,y)\,dx\ dy$

QUESTION 8

a) Let $x = r\cos\theta,\ \ y = r\sin\theta.$ Find the Jacobian $J = \dfrac{\partial(x,y)}{\partial(r,\theta)}$ of this transformation.

b) (i) Sketch the curve with rectangular equation $y = \sqrt{4-x^2}$. Rewrite the equation in polar form.

(ii) Rewrite the double integral $\int_0^2\int_0^{\sqrt{4-x^2}}(x^2+y^2)^3\ \ dy\,dx$ in polar coordinates and evaluate it.

c) (i) Sketch the curve with rectangular equation $y = \sqrt{2 - x^2}$ .Rewrite the equation in polar form.

(ii) Rewrite the double integral $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{0}^{\sqrt{2-x^2}} \frac{x}{x^2 + y^2} \, dy \, dx$ in polar coordinates and evaluate it.

d) (i) Sketch the curve with rectangular equation $x = \sqrt{1 - y^2}$. Rewrite the equation in polar form.

(ii) Rewrite the double integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \sqrt{x^2 + y^2} \, dx \, dy$ in polar coordinates and evaluate it.