AMAT 223 TEST 2 REVIEW PROBLEMS SPRING 2009

QUESTION 1

Let $\vec{F}(x,y)$ and $\vec{G}(x,y)$ be the following two vector fields:

$\vec{F}(x,y,z) = (xy^2)\ \hat{i} + (3y^4z)\ \hat{j} + (5z)\hat{k}$, $\vec{G}(x,y) = (zx^3)\ \hat{i} + (3y^2x)\ \hat{j} + (7x)\hat{k}$,

a) Find $\vec{\nabla}\times\vec{F}$ and $\vec{\nabla}\times\vec{G}$ b) Find $\vec{\nabla}\bullet\vec{F}$ and $\vec{\nabla}\bullet\vec{G}$

QUESTION 2

Consider the following vector fields:

$\vec{F}(x,y) = (2xy^3)\ \hat{i} + (1+3x^2y^2)\ \hat{j}$, $\vec{G}(x,y) = (y^2)\ \hat{i} + (2xy)\ \hat{j}$,

$\vec{H}(x,y) = (3x)\ \hat{i} + (2x^2+4y)\ \hat{j}$, $\vec{L}(x,y) = (y-x)\ \hat{i} + (2x+y)\ \hat{j}$

a) Find $\vec{\nabla}\times\vec{F}$ and $\vec{\nabla}\bullet\vec{G}$

b) Is the given vector field $\vec{F}(x,y)$ conservative? If it is, find an $f(x,y)$ such that $\vec{F} = \vec{\nabla}f$.

c) Is the given vector field $\vec{G}(x,y)$ conservative? If it is, find a $g(x,y)$ such that $\vec{G} = \vec{\nabla}g$.

d) Is the given vector field $\vec{H}(x,y)$ conservative? If it is, find a $h(x,y)$ such that $\vec{H} = \vec{\nabla}h$.

e) Is the given vector field $\vec{L}(x,y)$ conservative? If it is, find a $l(x,y)$ such that $\vec{L} = \vec{\nabla}l$.

f) Let C_1 be the curve $y = x^4$ from the point $(0,0)$ to the point $(1,1)$. Evaluate the line integral $\int_{C_1} 3x\ dx + (2x^2+4y)\ dy$.

g) Let C_2 be the curve $y = x^2$ from the point $(0,0)$ to the point $(2,4)$. Evaluate the line integral $\int_{C_2} (y-x)\ dx + (2x+y)\ dy$.

h) Let C_3 be the curve $y = x^2+3x+1$ from the point $(0,1)$ to the point $(1,5)$. Evaluate the line integral $\int_{C_3} 2xy^3\,dx + (1+3x^2y^2)\,dy$.

i) Let C_4 a piecewise defined curve from the point (0,0) to the point $(3,4)$. Evaluate the line integral $\int_{C_4} y^2\ dx + 2xy\,dy$

QUESTION 3

a) Let $f(x, y)$ be a function for which the following double integrals exist.

Rewrite each double integral with the order of integration reversed:

(i) $\int_{7}^{22} \int_{12}^{34} f(x,y)\ dy\ dx$ (ii) $\int_{0}^{3} \int_{y/3}^{1} f(x,y)\ dx\ dy$

(iii) $\int_{0}^{4} \int_{1}^{e^x} f(x,y)\ dy\ dx$ (iv) $\int_{0}^{4} \int_{\sqrt{y}}^{2} f(x,y)\ dx\ dy$

b) Evaluate the following integrals reversing the order of integration:

(i) $\int_{1}^{3} \int_{0}^{\ln x} x\, dy\ dx$ (ii) $\int_{0}^{1} \int_{x}^{1} e^y dy\ dx$

QUESTION 4

a) Let $x = r\cos\theta,\ \ y = r\sin\theta.$ Find the Jacobian $J = \dfrac{\partial(x,y)}{\partial(r,\theta)}$ of this transformation.

b) (i) Sketch the curve with rectangular equation $y = \sqrt{9 - x^2}$. Rewrite the equation in polar form.

(ii) Rewrite the double integral $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} (x^2 + y^2)^2 \quad dy\ dx$ in polar coordinates and evaluate it.

c) (i) Sketch the curve with rectangular equation $y = \sqrt{3 - x^2}$ .Rewrite the equation in polar form.

(ii) Rewrite the double integral $\int_{0}^{\sqrt{3}} \int_{0}^{\sqrt{3-x^2}} \dfrac{y}{x^2 + y^2}\ dy\ dx$ in polar coordinates and evaluate it.

d) (i) Sketch the curve with rectangular equation $x = \sqrt{1 - y^2}$ .Rewrite the equation in polar form.

(ii) Rewrite the double integral $\int_{0}^{1} \int_{0}^{\sqrt{1-y^2}} \sqrt{x^2 + y^2}\ \ dx\ dy$ in polar coordinates and evaluate it.