



**QUESTION 1**

Find the general solution to each of the following differential equations:

a)  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0.$

d)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0.$

b)  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0.$

e)  $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 0$

c)  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0.$

f)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0$

**QUESTION 2**

I. a) Find the general solution to the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0.$

b) Use part (a) and the method of *undetermined coefficients* to find the general solution to the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = e^{-x}.$

c) Use part (b) to find the unique solution of the above differential equation with initial conditions  $y(0) = 0, y'(0) = 1.$

d) Use part (a) and the method of *undetermined coefficients* to find the general solution to the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 3x.$

e) Find the form of the particular solution of the above differential equation if the right hand side of the above equation is:

- i)  $x^2$       ii)  $e^{4x}$       and      iii)  $\cos 2x.$

II. a) Find the general solution to the differential equation  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0.$

b) Use part (a) and the method of *undetermined coefficients* to find the general solution to the differential equation  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^x.$

c) Find the form of the particular solution of the above differential equation if the right hand side of the above equation is:

- i)  $5x$       ii)  $e^{2x}$       iii)  $\cos 2x$       iv)  $e^{-2x}$

**QUESTION 3**

a) Use the method of variation of parameters to find the general solution to the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^x$$

b)  $y_1(x) = 1 + x$  and  $y_2(x) = e^x$  are solutions to the differential equation

$$\frac{d^2 y}{dx^2} - \frac{(1+x)}{x} \cdot \frac{dy}{dx} + \frac{y}{x} = 0.$$

Use the method of variation of parameters to find the general solution to the differential equation

$$\frac{d^2 y}{dx^2} - \frac{(1+x)}{x} \frac{dy}{dx} + \frac{y}{x} = xe^{2x}.$$

#### QUESTION 4

a) Use the Power Series method to find the solution to the differential equation

$$\frac{d^2 y}{dx^2} + (x+1) \cdot \frac{dy}{dx} + (2x-1)y = 4x+2$$

with the initial conditions  $y(0) = 1$  and  $y'(0) = 0$ , up to and including the term in  $x^4$ .  
Approximate  $y(0.5)$ .

b) Use the Power Series method to find the solution to the differential equation

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$$

with the initial conditions  $y(0) = 1$  and  $y'(0) = 0$ , up to and including the term in  $x^4$ .

#### QUESTION 5

a) Use the method of Laplace transforms to solve the differential equation:

$$\frac{d^2 y}{dx^2} + 8 \frac{dy}{dx} + 7y = 0 \text{ with } y(0) = 1 \text{ and } y'(0) = 1.$$

b) Use the method of Laplace transforms to solve the differential equation:

$$\frac{d^2 y}{dx^2} + 16y = 1 \text{ with } y(0) = 0 \text{ and } y'(0) = 1.$$