FREDERICK UNIVERSITY CYPRUS

AMAT 204 SPRING 2009 PRACTICE PROBLEMS FOR TEST 1

QUESTION 1

Solve the following differential equations.

a) $\frac{dy}{dx} = \frac{4x^3}{y}$; $y(0) = -2$.
Give the unique solution in explicit form.

b) $\frac{dy}{dx} = \frac{e^x}{y}$; $y(0) = 2$.
Give the unique solution in explicit form.

c) $\frac{dy}{dx} = 5y$; $y(0) = 10$.
Give the unique solution in explicit form.

d) $\frac{dy}{dx} - 2xy = -x$. Find an integrating factor first.
Give the general solution in explicit form.

QUESTION 2

Solve the following differential equations:

a) $\frac{dy}{dx} = \frac{y^2 + 3xy}{x^2}$.
(Hint : Use the substitution $u = \frac{y}{x}$).

b) $\frac{dy}{dx} + 2y = x$. Find an integrating factor first.
Give the general solution in explicit form.

c) $\frac{dy}{dx} = \frac{4y + 3x}{x}$.
(Hint : Use the substitution $u = \frac{y}{x}$).

d) $x\frac{dy}{dx} + 3y = x^2$. Find an integrating factor first.
Give the general solution in explicit form.

QUESTION 3

Consider the differential equation $(1 + x^2) \cdot \frac{dy}{dx} + 2xy = 0$:

a) Show that it is exact.
b) Solve it, giving the general solution in an implicit form.
c) Give the general solution in an explicit form, as well.
d) Given that $y(0) = 1$, find the unique solution.

QUESTION 4

Consider the differential equation $(3y^2 - 6xy) \cdot \frac{dy}{dx} - (2x + 3y^2) = 0$:

a) Show that it is exact.
b) Solve it, giving the general solution in an implicit form.
c) Given that $y(0) = 2$, find the unique solution.

QUESTION 5

Consider the differential equation $2xy \cdot \frac{dy}{dx} + (2y^2 + 3x) = 0$:

a) Show that it is <u>not</u> exact.
b) Use the integrating factor $\mu = x$ to obtain a new differential equation. Show that this one is exact.
c) Solve it, giving the general solution in an implicit form.
d) Give the general solution in an explicit form, as well.

QUESTION 6

Find the general solution to each of the following differential equations:

a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$.

b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.

c) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - 8y = 0$.

d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0$.

e) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 0$

f) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$