**POWER SCREWS (ACME THREAD) DESIGN**

There are at least three types of power screw threads: the square thread, the Acme thread, and the buttress thread. Of these, the square and buttress threads are the most efficient. That is, they require the least torque to move a given load along the screw. However, the Acme thread is not greatly less efficient, and it is easier to machine. The buttress thread is desirable when force is to be transmitted in only one direction.

As with fasteners, manufacturers have developed preferred combinations of basic major diameter, $D$, and number of threads per inch, $n$, for Acme screw threads. The pitch, $p$, is the distance from a point on one thread to the corresponding point on the adjacent thread, and $p = 1/n$.

Other pertinent dimensions documented include the minimum minor diameter and the minimum pitch diameter of a screw with an external thread. When you are performing stress analysis on the screw, the safest approach is to compute the area corresponding to the minor diameter for tensile or compressive stresses. However, a more accurate stress computation results from using the *tensile stress area*, found from

$$A_t = \frac{\pi}{4} \left[ \frac{D_r + D_p}{2} \right]^2$$

This is the area corresponding to the average of the minor (or root) diameter, $D_r$, and the pitch diameter, $D_p$. The data reflect the minimums for commercially available screws according to recommended tolerances.

The shear stress area, $A_s$, is also found in published data and represents the area in shear approximately at the pitch line of the threads for a 1.0-in length of engagement. Other lengths would require that the area be modified by the ratio of the actual length to 1.0 in.

When using a power screw to exert a force, as with a jack raising a load, you need to know how much torque must be applied to the nut of the screw to move the load. The parameters involved are the force to be moved, $F$; the size of the screw, as indicated by its pitch diameter, $D_p$; the lead of the screw, $L$; and the coefficient of friction, $f$. Note that the *lead* is defined as the axial distance that the screw would move in one complete revolution. For the usual case of a single-threaded screw, the lead is equal to the pitch and for the usual case of the single-threaded screw can be computed from

$$L = \frac{p}{1/n}$$

The torque to move a load up the thread is

$$T_u = \frac{F \cdot D_p}{2} \left[ \frac{L + \pi \cdot f \cdot D_p}{\pi \cdot D_p - f \cdot L} \right]$$

This equation accounts for the force required to overcome friction between the screw and the nut in addition to the force required just to move the load. If the screw or the nut bears against a stationary surface while rotating, there will be an additional friction torque developed at that surface. For this reason, many jacks and similar devices incorporate antifriction bearings at such points.

The coefficient of friction $f$ depends on the materials used and the manner of lubricating the screw. For well-lubricated steel screws acting in steel nuts, $f = 0.15$ should be conservative.

An important factor in the analysis for torque is the angle of inclination of the plane. In a screw thread, the angle of inclination is referred to as the *lead angle*, $\lambda$. It is the angle between the
tangent to the helix of the thread and the plane transverse to the axis of the screw.

\[ \tan \lambda = \frac{L}{\pi \cdot D_p} \]

where \( \pi \cdot D_p \) = circumference of the pitch line of the screw

Then if the rotation of the screw tends to raise the load (move it up the incline), the friction force opposes the motion and acts down the plane.

Conversely, if the rotation of the screw tends to lower the load, the friction force will act up the plane. The torque analysis changes:

\[ T_d = \frac{F \cdot D_p}{2} \left[ \frac{\pi \cdot f \cdot D_p - L}{\pi \cdot D_p + f \cdot L} \right] \]

If the screw thread is very steep (that is, if it has a high lead angle), the friction force may not be able to overcome the tendency for the load to "slide" down the plane, and the load will fall due to gravity. In most cases for power screws with single threads, however, the lead angle is rather small, and the friction force is large enough to oppose the load and keep it from sliding down the plane. Such a screw is called self-locking, a desirable characteristic for jacks and similar devices. Quantitatively, the condition that must be met for self-locking is

\[ f > \tan \lambda \]

The coefficient of friction must be greater than the tangent of the lead angle. For \( f = 0.15 \), the corresponding value of the lead angle is 8.5°. For \( f = 0.1 \), for very smooth, well-lubricated surfaces, the lead angle for self-locking is 5.7°.

Efficiency for the transmission of a force by a power screw can be expressed as the ratio of the torque required to move the load without friction to that with friction. Letting \( f = 0 \), the torque required without friction, \( T' \), is

\[ T' = \frac{F \cdot L}{2 \cdot \pi} \]

Then the efficiency, \( e \), is

\[ e = \frac{T'}{T_u} = \frac{F \cdot L}{2 \cdot \pi \cdot T_u} \]

If the torque required to rotate the screw is applied at a constant rotational speed, \( n \), then the power in horsepower to drive the screw is

\[ P = \frac{T \cdot n}{63000} \]

The required tensile stress area for a screw loaded in tension is

\[ A_t = \frac{F}{S_{ut}} \]

where \( F \) = load to be moved; \( S_{ut} \) = tensile strength limit.
A screw in some applications carries a load transverse to its axis. Such loads produce bending moments in the screw, which result in the development of bending stresses. To determine the required section modulus, $S$, for such an application

$$S = \frac{M_{\text{max}}}{S_u}$$

Long power screws loaded in compression can fail by elastic instability as given by the Euler formula

$$F = \frac{C \cdot \pi^2 \cdot E}{A \left(\frac{b}{r}\right)^2}$$

where $F$ = total load, $A$ = area of section, $E$ = modulus of elasticity, $b/r$ = slenderness ratio, and $C$ is the coefficient of constraint, which depends on end conditions. For round ends, $C = 1$; for fixed ends, $C = 4$; and for the end conditions that occur in practice, $C$ can rarely be assumed greater than 2.

For power screws that behave as short columns, it is not possible to calculate with accuracy the maximum stress produced by a nominally concentric load because of the large influence of the indeterminate crookedness and eccentricity. The maximum unit stress that a column will sustain, however, can be expressed by any of a number of formulas, each of which contains one or more terms that is empirically adjusted to secure conformity with test results. Of such formulas, those given below are the best known and provide the basis for most of the design formulas used in American practice. In these equations $F$ denotes the load at failure, $A$ the cross-sectional area, $b$ the length, and $r$ the least radius of gyration of the section; the meaning of other symbols used is explained in the discussion of each formula.

Rankine formula

$$\frac{P}{A} = \frac{\sigma}{1 + \phi \left(\frac{L}{r}\right)^2}$$

This is a semirational formula. The value of $\sigma$ is sometimes taken as the ultimate strength of the material and the value of $\phi$ as $\sigma / C \cdot \pi^2 \cdot E$, thus making the formula agree with the results of tests on short prisms when $L/r$ is very small and with Euler's equation when $L/r$ is very large. More often $\sigma$ and $\phi$ are adjusted empirically to make the equation agree with the results of tests through the $L/r$ range of most importance.

**Modulus of Elasticity in Tension**

For the part of the stress-strain diagram that is straight, stress is proportional to strain, and the value of $E$ is the constant of proportionality. That is,

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon}$$

This is the slope of the straight-line portion of the diagram. The modulus of elasticity indicates the stiffness of the material, or it's resistance to deformation. To choose a value in this software, simply click on the database icon and find the material for your application.
**Tensile strength**

The tensile strength of a material, \( S_u \), and \( S_y \) is determined from test results, supplier specifications, or given data. The most accurate and reliable data available should be used. When there is doubt about the accuracy of the data, larger-than-average design factors should be used.

For the repeated, reversed bending in a shaft caused by transverse loads applied to the rotating shaft, the ultimate tensile strength is related to the endurance strength of the shaft material. The actual conditions under which the shaft is manufactured and operated should be considered when specifying the design stress.

**Shear strength**

The strength in shear \( S_S \) of a material is an important property. Unfortunately, these values are seldom reported and we use the following estimates to determine \( S_S \)

\[
S_{us} = 0.75 \cdot S_u \\
S_{ys} = 0.5 \cdot S_y
\]

where \( S_u \) and \( S_y \) are reported as tensile properties.

**Design factor**

Under typical industrial conditions, the design factor of \( N = 3 \) is recommended. If the application is very smooth, a value as low as \( N = 2 \) may be justified. Under conditions of shock or impact, \( N = 4 \) or higher should be used, and careful testing is advised.

**Coefficient of friction, \( f \)**

The coefficient of friction for use in analyzing ball screws depends on the materials used and the manner of lubricating the screw. For well-lubricated steel screws acting in steel nuts, \( f = 0.15 \) should be conservative.

While it has long been the custom to assume a value of \( f = 0.15 \) as a reasonable expectation, more recently, Ham and Ryan have conducted a group of experiments and found that if the threads are cut accurately, if the thread surfaces are smooth, and if they are well lubricated, the coefficient of friction may be expected to average as low as 0.10. Messrs. Ham and Ryan recommend that for an average quality of materials and workmanship, the coefficient of friction in motion be taken as 0.125; while for inferior workmanship and quality of materials, the coefficient of friction may be assumed as 0.15. Increase these values 30 to 35 per cent for starting friction.

Additionally the research indicates that the coefficient of friction is practically independent of the axial load; it undergoes negligible changes due to speed for the speed ranges encountered in practice; it decreases somewhat as the lubricants used become heavier; it shows little variation for different combinations of material, being lowest for soft steel on bronze, and hardened and ground steel on bronze; and it is in close agreement with the results obtained by the theoretical equations.
Typical Acme screw-driven system

Fig. 1

Screw thread force analysis

\[
P = \text{Force required to move the load} \\
F_f = \text{Friction force} \\
N = \text{Normal force} \\
\lambda = \text{Lead angle} \\
D_p = \text{Pitch diameter}
\]

Fig. 2
Typical stress-strain diagram

Diagram for steel

Diagram for aluminium and other metals having no yield point

Fig. 3
**MDESIGN SOFTWARE - Example Power Screw Design**

**Input data:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Modulus of elasticity E</td>
<td>206 842 MPa</td>
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<tr>
<td>Tensile strength limit Su</td>
<td>68.94 MPa</td>
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<td>Shear strength limit Sus</td>
<td>34.47 MPa</td>
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<td>Design Factor N</td>
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<tr>
<td>Number of screws NS</td>
<td>2</td>
</tr>
<tr>
<td>Load to be moved F</td>
<td>1 112 05 N</td>
</tr>
<tr>
<td>Action distance S</td>
<td>381 mm</td>
</tr>
<tr>
<td>Time to pass the action distance t</td>
<td>12 s</td>
</tr>
<tr>
<td>Thread angle φ</td>
<td>14.5°</td>
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<tr>
<td>Coefficient of friction f</td>
<td>0.15</td>
</tr>
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</table>

**Results:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Nominal major diameter D</td>
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<tr>
<td>Pitch P</td>
<td>6.35 mm</td>
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<tr>
<td>Minimum minor diameter Dr</td>
<td>30.39 mm</td>
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<tr>
<td>Minimum pitch diameter Dp</td>
<td>34.11 mm</td>
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<tr>
<td>Tensile stress area At</td>
<td>816.57 mm²</td>
</tr>
<tr>
<td>Shear stress area per inch of engagement As</td>
<td>1509.94 mm²</td>
</tr>
<tr>
<td>Minimal required length of engagement h</td>
<td>27.125 mm</td>
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<tr>
<td>Torque required to raise the load Tu</td>
<td>205.02 Nm</td>
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<tr>
<td>Torque required to lower the load Td</td>
<td>89.91 Nm</td>
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<tr>
<td>Efficiency e</td>
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<td>Linear speed V</td>
<td>0.032 m/s</td>
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<tr>
<td>Rotational speed n</td>
<td>300 rpm</td>
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<tr>
<td>Power required to drive each screw P1</td>
<td>6 440 W</td>
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<tr>
<td>Total power P</td>
<td>1 288 W</td>
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