Biologically Inspired Bearing-Only Navigation and Tracking

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***Abstract*— In this paper we develop controllers that are used for the control of individual or groups of vehicles based only on sensors that provide bearing information. Our inspiration is derived from the observation that many ant species use landmark retinal positions to navigate without having any range information. This is specially relevant to vision-based controllers for vehicles because cameras provide very good bearing information but relatively poor range information. We present a provably correct bearing-only navigation controller and a methodology for tracking that lends itself to control of formations. The proposed feedback controllers are shown to have analytically guaranteed properties. The effectiveness of the proposed controllers is demonstrated through computer simulations.**

I. Introduction

Biological systems have adapted through their evolution to be capable of utilizing minimal resources, in terms of information and energy, to carry out their every day tasks. This fact is one of the major reasons the scientific community is trying to derive inspiration from biological systems and then apply them to artificial systems.

The capability that inspired us towards this line of research is the one of being able to navigate without any global positioning system, without any range measurement sensors but with only the retinal positions of landmarks. The desert ants *Cataglyphis* are capable of traveling thousands of times their body lengths to finally arrive at a pinpoint goal [7]. In contrast to most other ant species, they do not use pheromones to mark their path, but use a combination of strategies including path integration, visual landmarks and the sunlight polarization vector to carry out this task. Experiments with emigrating workers of *Leptothorax albipennis* have shown that ants use visual landmarks as a beacon when navigating between sites even with very small eyes (containing only about 60 ommatidia) [5].

These observations have triggered research directed in constructing artificial systems capable of navigation by mimicking the behavior observed in ants. In [3] based on experiments performed on bees, a model of matching a snapshot and the current view, the "snapshot matching model" was proposed for landmark based navigation, where radial and tangential contributions of retinal images were accounted to produce the motion strategy. In [6] the authors, inspired by the navigation strategies implemented by the ant *Cataglyphis*, constructed mechanisms for path integration and visual piloting that were successfully employed on an autonomous mobile robot. They implemented a variation of the "snapshot model" in conjunction with a polarization compass for landmark based navigation. In [4] the authors propose a homing scheme that uses parameterized displacement fields obtained from an approximation that incorporates prior knowledge about perspective distortions of the visual environment. More recently in [1], [2] the authors propose a combination of two control laws implemented under a hybrid systems framework to achieve bearing only navigation.

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In this paper we propose a novel technique for robotic navigation and tracking, that requires only bearing information from at least three unknown landmarks. Although bearing -only techniques have been proposed in the past, this is the first technique, to our knowledge, possessing analytical guarantees on its navigation and tracking properties. The major contributions of this paper can be summarized as follows:

- A provably correct way to navigate and stabilize the system using only bearing measurements from three, unknown landmarks
- A provably correct way to implement bearing-only measurements for tracking a reference point in landmark coordinates, when the landmarks are non-stationary.

The rest of the paper is organized as follows: Section II discusses some preliminary notions and presents our solution to the bearing only navigation problem. Section III defines the considered tracking problem and proposed a tracking controller design based on the navigation controller. Section IV presents several computer simulations of the proposed navigation and tracking controllers and the paper concludes with section V.

II. Bearing Only Navigation

A. Preliminaries

Let $x \in \mathbb{E}^2$ be the state of the system. We will assume for our analysis that we have three landmarks, which is the minimum number of landmarks required to localize a robot with bearing information only. Let $A, B, C \in \mathbb{E}^2$ be the landmark points. Then $a, b, c \in \mathbb{E}^2$ are the position vectors of the landmarks with respect to the current system location (see figure 1). We will use the notation Ξ and ξ to refer to landmark positions and landmark vectors respectively. By definition $\xi = \Xi - x$, with $\{\xi, \Xi\} \in \{\{a, A\}, \{b, B\}, \{c, C\}\}$. In the following analysis whenever we write ξ we will imply any landmark vector and with Ξ the corresponding landmark position. The bearing measurements are represented with the with the vectors $\hat{\xi}$ and are defined as follows:

$$\hat{\xi} \triangleq \frac{\xi}{\|\xi\|}$$

. The triple $\left(\hat{a}, \hat{b}, \hat{c}\right)$ defines a coordinate system that we will refer to as the *bearing coordinates*. Note that if the landmarks are non collinear, any $x \in \mathbb{E}^2$ can be described using the bearing coordinates as shown in Figure 1.

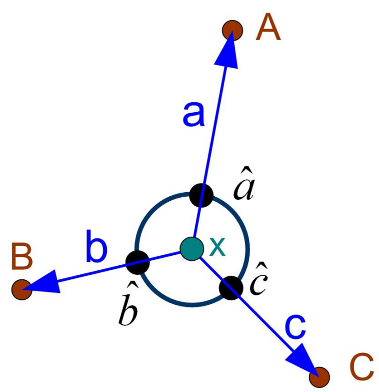

Fig. 1 *: Landmarks A, B, C, agent's position x and bearing measurements $\hat{a}, \hat{b}, \hat{c}$*

B. Problem Statement

Consider a system described through a kinematic model:

$$\dot{x} = u \tag{1}$$

More specifically, the problem that we are considering in this section can be formally stated as follows.

> *Given system (1), a workspace with three non-collinear static landmarks whose location and geometry are not known, derive a feedback control scheme that will drive the system from any initial location to the destination location given by its bearing coordinates, using only the bearing measurements corresponding to the landmarks.*

C. Controller Design

Our approach in designing the controller was to use the perpendicular vectors to the bearing measurements to create the navigation vector field.

Let $(\hat{a}^*, \hat{b}^*, \hat{c}^*)$ denote the destination bearing coordinates. Define the perpendicular operator J as follows:

$$J \triangleq \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

The perpendicular bearing vectors are then defined as:

$$\widehat{\xi}^{\perp} \triangleq J\widehat{\xi}$$

When we are on a landmark and $\hat{\xi}$ is not defined, we define it to be zero. We have the following result:

Proposition 1: Suppose that $\hat{a}^*, \hat{b}^*, \hat{c}^*$ are valid destination bearings. Then system 1 under the control law:

$$u = -\sum_{\xi \in \{a,b,c\}} \left(\widehat{\xi}^{*T} \widehat{\xi}^{\perp}\right) \widehat{\xi}^{\perp} \tag{2}$$

is globally asymptotically stable.

Proof: Consider the following Lyapunov function candidate:

$$V = \sum_{\xi \in \{a,b,c\}} \left(1 - \widehat{\xi}^{*T} \widehat{\xi}\right)^2 \tag{3}$$

Evaluating the time derivative, we have

$$\frac{\dot{V}}{2} = -\sum_{\xi \in \{a,b,c\}} \left(1 - \widehat{\xi}^{*T} \widehat{\xi}\right) \widehat{\xi}^{*T} \dot{\widehat{\xi}}$$

Considering the definitions of ξ, $\hat{\xi}$ and the fact that the landmarks are stationary, the time derivative of $\dot{\hat{\xi}}$ is evaluated as:

$$\dot{\widehat{\xi}} = \frac{1}{\|\xi\|} \left(\widehat{\xi}\widehat{\xi}^T - I\right) \dot{x}$$

Substituting in $\dot{V}$, we get:

$$\frac{\dot{V}}{2} = -\sum_{\xi \in \{a,b,c\}} \frac{1}{\|\xi\|} \left(1 - \widehat{\xi}^{*T} \widehat{\xi}\right) \left(\widehat{\xi}^{*T} \widehat{\xi}\widehat{\xi}^T - \widehat{\xi}^{*T}\right) \dot{x}$$

Substituting the control law we get:

$$\frac{\dot{V}}{2} = \sum_{\xi \in \{a,b,c\}} \frac{1}{\|\xi\|} \left(1 - \widehat{\xi}^{*T} \widehat{\xi}\right) \left(\widehat{\xi}^{*T} \widehat{\xi}\widehat{\xi}^T - \widehat{\xi}^{*T}\right) \cdot$$

$$\sum_{\xi \in \{a,b,c\}} \left(\widehat{\xi}^{*T} \widehat{\xi}^{\perp}\right) \widehat{\xi}^{\perp}$$

Breaking the sum in two parts, one containing the squared terms and one containing the cross terms, we get:

$$\frac{\dot{V}}{2} = -\sum_{\xi \in \{a,b,c\}} \frac{1}{\|\xi\|} \left(1 - \widehat{\xi}^{*T} \widehat{\xi}\right) \left(\widehat{\xi}^{*T} \widehat{\xi}^{\perp}\right)^2 +$$

$$\sum_{\substack{\{\mu,\nu\} \in \{a,b,c\} \\ \mu \neq \nu}} \frac{\left(1 - \widehat{\mu}^{*T} \widehat{\mu}\right) \left(\widehat{\mu}^{*T} \widehat{\mu}\widehat{\mu}^T \widehat{\nu}^{\perp} - \widehat{\mu}^{*T} \widehat{\nu}^{\perp}\right) \left(\widehat{\nu}^{*T} \widehat{\nu}^{\perp}\right)}{\|\mu\|}$$

We have that:

$$\widehat{\mu}^{*T} \widehat{\mu}\widehat{\mu}^T \widehat{\nu}^{\perp} - \widehat{\mu}^{*T} \widehat{\nu}^{\perp} = \widehat{\mu}^{*T} \left(\widehat{\mu}\widehat{\mu}^T - I\right) \widehat{\nu}^{\perp}$$

But $\widehat{\mu}\widehat{\mu}^T - I = -\widehat{\mu}^{\perp}\widehat{\mu}^{\perp T}$ since it is a projector. So we can write:

$$\frac{\dot{V}}{2} = -\sum_{\xi \in \{a,b,c\}} \frac{1}{\|\xi\|} \left(1 - \widehat{\xi}^{*T} \widehat{\xi}\right) \left(\widehat{\xi}^{*T} \widehat{\xi}^{\perp}\right)^2$$

$$-\sum_{\substack{\{\mu,\nu\} \in \{a,b,c\} \\ \mu \neq \nu}} \frac{1}{\|\mu\|} \left(1 - \widehat{\mu}^{*T} \widehat{\mu}\right) \left(\widehat{\mu}^{*T} \widehat{\mu}^{\perp} \widehat{\mu}^{\perp T} \widehat{\nu}^{\perp}\right) \left(\widehat{\nu}^{*T} \widehat{\nu}^{\perp}\right)$$

Note in the above that $\widehat{\mu}^{\perp T} \widehat{\nu}^{\perp} = \widehat{\mu}^T \widehat{\nu}$. We can now write the above as the following sum of quadratic forms:

$$\frac{\dot{V}}{2} = -\sum_{\substack{\{\mu,\nu\}\in\{a,b,c\} \\ \nu>\mu}} \begin{bmatrix} \widehat{\mu}^{*T}\widehat{\mu}^{\perp} \\ \widehat{\nu}^{*T}\widehat{\nu}^{\perp} \end{bmatrix}^T M_{\mu,\nu} \begin{bmatrix} \widehat{\mu}^{*T}\widehat{\mu}^{\perp} \\ \widehat{\nu}^{*T}\widehat{\nu}^{\perp} \end{bmatrix}$$

Note here that in $\nu > \mu$ the inequality sign is somehow abused to indicate that permutations between the indices are not allowed and $\nu \neq \mu$. The matrix

$$M_{\mu,\nu} = \begin{bmatrix} \frac{1}{\|\mu\|}\left(1-\hat{\mu}^{*T}\hat{\mu}\right) & \frac{\hat{\mu}^T\hat{\nu}}{\|\mu\|}\left(1-\hat{\mu}^{*T}\hat{\mu}\right) \\ \frac{\hat{\nu}^T\hat{\mu}}{\|\nu\|}\left(1-\hat{\nu}^{*T}\hat{\nu}\right) & \frac{1}{\|\nu\|}\left(1-\hat{\nu}^{*T}\hat{\nu}\right) \end{bmatrix}.$$

The eigenvalues of $M_{\mu,\nu}$ are:

$$\lambda_{1,2} = \frac{\left(\|\mu\|\left(1-\hat{\nu}^{*T}\hat{\nu}\right)+\|\nu\|\left(1-\hat{\mu}^{*T}\hat{\mu}\right)\pm r\right)}{2\,\|\nu\|\,\|\mu\|}$$

where

$$\begin{aligned} r^2 \;=\;& 4\,\|\mu\|\,\|\nu\|\left(1-\hat{\mu}^{*T}\hat{\mu}\right)\left(1-\hat{\nu}^{*T}\hat{\nu}\right)\left(\left(\hat{\mu}^T\hat{\nu}\right)^2-1\right) \\ &+\left(\|\mu\|\left(1-\hat{\nu}^{*T}\hat{\nu}\right)+\|\nu\|\left(1-\hat{\mu}^{*T}\hat{\mu}\right)\right)^2 \end{aligned}$$

Note that since $\left(\hat{\mu}^T\hat{\nu}\right)^2 \leq 1$ then

$$\left(\|\mu\|\left(1-\hat{\nu}^{*T}\hat{\nu}\right)-\|\nu\|\left(1-\hat{\mu}^{*T}\hat{\mu}\right)\right)^2 \leq r^2$$

and

$$r^2 \leq \left(\|\mu\|\left(1-\hat{\nu}^{*T}\hat{\nu}\right)+\|\nu\|\left(1-\hat{\mu}^{*T}\hat{\mu}\right)\right)^2$$

Thus the eigenvalues of $M_{\mu,\nu}$ are bounded as follows:

$$0 \leq \lambda_1 \leq \lambda_2 \leq \frac{\|\mu\|\left(1-\hat{\nu}^{*T}\hat{\nu}\right)+\|\nu\|\left(1-\hat{\mu}^{*T}\hat{\mu}\right)}{\|\nu\|\,\|\mu\|}$$

Whenever $\lambda_1(M_{\mu,\nu}) = 0$, this implies that at least one of the following is true: $\hat{\nu} = \pm\hat{\mu}$, $\hat{\nu} = \hat{\nu}^*$, $\hat{\mu} = \hat{\mu}^*$. When $\lambda_2(M_{\mu,\nu}) = 0$, it implies that both $\hat{\nu} = \hat{\nu}^*$ and $\hat{\mu} = \hat{\mu}^*$ are true.

If all $\hat{\xi}$ are away from their targets and $\lambda_1(M_{\mu,\nu}) = 0$, this necessarily implies that $\hat{\nu} = \pm\hat{\mu}$. If for some $\hat{\xi} \neq \{\mu,\nu\}$ it is true that $\lambda_1(M_{\mu,\xi}) = 0$ then this would imply $\hat{\xi} = \pm\hat{\mu}$, which combined with the above implies the collinearity of landmarks, violating the non-collinearity assumption. Hence we will have at least one $M_{\mu,\xi} > 0$. If $\hat{\nu} = \hat{\nu}^*$ then $M_{\nu,\xi} = \begin{bmatrix} 0 & 0 \\ \frac{\hat{\xi}^T\hat{\nu}}{\|\xi\|}\left(1-\hat{\xi}^{*T}\hat{\xi}\right) & \frac{1}{\|\xi\|}\left(1-\hat{\xi}^{*T}\hat{\xi}\right) \end{bmatrix}$, thus its related quadratic form will be positive since $\hat{\xi} \neq \hat{\xi}^*$. Using the same reasoning for the case when $\{\hat{\mu},\hat{\nu}\} = \{\hat{\mu}^*,\hat{\nu}^*\}$ we conclude that $\dot{V} = 0$ only when all $\{\hat{a},\hat{b},\hat{c}\} = \{\hat{a}^*,\hat{b}^*,\hat{c}^*\}$. Hence we have that:

$$\dot{V} < 0$$

and the system is globally asymptotically stable. ■

III. TRACKING

A. Problem statement

In this section we relax the assumption that landmarks are static. The target is defined as the location corresponding to the destination bearing coordinates. In order for the destination bearing coordinates to be valid, the landmarks are constrained to move such that the destination bearing coordinates corresponds to a location in the workspace.

More specifically the problem that we are considering in this section can be formally stated as follows:

> Assume system (1), a workspace with three non-collinear landmarks whose location and geometry are not known, and a target. The landmarks are allowed to move such that the target bearing coordinates remain valid. Derive a feedback based control scheme that will track the trajectory of the target using only bearing measurements corresponding to the landmarks.

B. Controller Design

Our approach in designing the tracking controller is to use controller (2) and an appropriate controller for regulating the gain. We have the following result:

Proposition 2: Assume that $\|\dot{\Xi}\|$ and $\|\ddot{\Xi}\|$ are upper bounded and the target bearing coordinates are valid. Then system (1) under the control law

$$u = -k\sum_{\xi\in\{a,b,c\}}\left(\hat{\xi}^{*T}\hat{\xi}^{\perp}\right)\hat{\xi}^{\perp} \qquad (4)$$

$$\dot{k} = c_2^+\mathcal{H}\left(\frac{\dot{V}}{c_1}+V-\varepsilon\right) - c_2^-\mathcal{H}\left(-\frac{\dot{V}}{c_1}-V-\varepsilon\right)\mathcal{H}\left(k-1\right) \qquad (5)$$

where $\mathcal{H}$ is the Heaviside step function, ε, c_1 and $c_2^+ > c_2^-$ are positive constants, and V is the Lyapunov function defined in (3), converges asymptotically to an ε neighborhood of the destination bearing.

Proof: If we wanted our system to converge exponentially then we would require that:

$$\dot{V} = -c_1 V$$

. To this extend we define the error term:

$$e = \frac{\dot{V}}{c_1} + V$$

Since landmarks are not static, we have that:

$$\dot{\hat{\xi}} = \frac{1}{\|\xi\|}\left(\widehat{\xi}\widehat{\xi}^T - I\right)\left(\dot{x} - \dot{\Xi}\right)$$

Substituting in $\dot{V}$, we get:

$$\frac{\dot{V}}{2} = -\sum_{\xi\in\{a,b,c\}} g_\xi^T\left(\dot{x}-\dot{\Xi}\right)$$

where

$$g_\xi^T = \frac{1}{\|\xi\|}\left(1-\widehat{\xi}^{*T}\widehat{\xi}\right)\left(\widehat{\xi}^{*T}\widehat{\xi}\widehat{\xi}^T - \widehat{\xi}^{*T}\right)$$

Subscript ξ in g_ξ will be dropped to simplify notation. Writing the control law as $u = ku_0$, where u_0 is the control law defined in (2) and substituting in $\dot{V}$, we get:

$$\frac{\dot{V}}{2} = -k \sum g^T u_0 + \sum g^T \dot{\Xi}$$

Take the time derivative of the error term:

$$\dot{e} = \frac{\ddot{V}}{c_1} + \dot{V}$$

after substituting and regrouping terms, we get:

$$\dot{e} = -\frac{\dot{k}}{c_1} D - k\left(\frac{1}{c_1}\dot{D} + D\right) + \frac{1}{c_1}\dot{E} + E$$

where $D = 2\sum g^T u_0$, $E = 2\sum\left(g^T \dot{\Xi}\right)$.

Consider the Lyapunov function as $V = V(x, \Xi)$. Then the time derivative is $\dot{V} = \nabla_x V \dot{x} + \nabla_\Xi V \dot{\Xi}$. The first part $\nabla_x V \dot{x} = -kD$ is strictly negative for the control $\dot{x} = u_0$ as shown in the proof of Proposition refNavProp and it will also be negative for $\dot{x} = ku$ as long as $k > 0$. To determine the evolution of the error term, we construct the following Lyapunov function candidate:

$$V_2 = \frac{1}{2} e^T e$$

Taking the time derivative and after substitutions, we get:

$$\dot{V}_2 = -\frac{e}{c_1}\dot{k}D - \frac{e}{c_1}\left(k\left(\dot{D} + c_1 D\right) - \dot{E}\right) + eB$$

By assumption, $\|\dot{\Xi}\|$ and $\|\ddot{\Xi}\|$ are upper bounded by a constant but unknown quantity. Moreover assume that k is upper bounded by an increasing function of the upper bound of $\|\dot{\Xi}\|$. We will later show that this assumption is true as long as the initial condition for k is $k(0) = 1$. This implies that $D, \dot{D}, E, \dot{E}$ are upper bounded. Assume $F_{\max} = |k\left(\dot{D} + c_1 D\right) - \dot{E}|_{\max}$. Then

$$\dot{V}_2 \leq -\frac{e}{c_1}\dot{k}D + \frac{|e|}{c_1}F_{\max} + |e|E_{\max}$$

For $|e| \geq \varepsilon$ by design $\dot{k}$ will have the same sign as e. Then for all $|e| \geq \varepsilon$ we have that:

$$\dot{V}_2 < -|e|\left(\frac{c_2^-}{c_1}D - \frac{F_{\max}}{c_1} - E_{\max}\right)$$

Outside an ε neighborhood of the target we have $D \geq \delta(\varepsilon)$.

Choosing c_2^- such that

$$c_2^- > \frac{c_1}{\delta(\varepsilon)}\left(\frac{F_{\max}}{c_1} + E_{\max}\right)$$

, guarantees that for $|e| \geq \varepsilon$:

$$\dot{V}_2 < 0$$

i.e. the trajectories of our system away from an ε neighborhood of the target, converge asymptotically to the ε neighborhood of the trajectories of an asymptotically converging system, since they converge asymptotically to the manifold $\dot{V} \leq -c_1 V + \varepsilon$.

Regarding the assumption on k it can now be verified since the error e is converging asymptotically, it will enter the region $|e| < \varepsilon$ in finite time. Assuming that $\dot{\Xi}$ and $\ddot{\Xi}$ are bounded, then there is a minimum value of k_o such that $|e| < \varepsilon$ for all $k \geq k_o$. By the construction of the control law, this in turn implies that for such a k it will be $\dot{k} \leq 0$ and the proof is complete. ■

IV. Simulation results

To verify the effectiveness of our proposed controllers, we have prepared a set of simulations for the navigation and the tracking controller.

A. Navigation Simulations

In the first navigation simulation the landmarks were positioned on the vertices of an obtuse triangle. The system initiated from several initial conditions and the destination was inside the landmark triangle. Figure 2 depicts the trajectories of the system. As we can see the controller was successful in navigating the system.

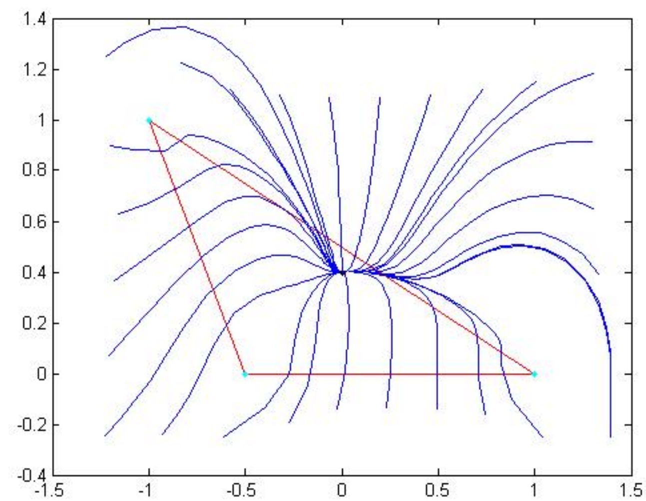

Fig. 2 *: Navigation: Flows of the system for various initial conditions with the destination inside the obtuse landmark triangle*

In the second navigation simulation, the landmarks were positioned on the vertices of an acute triangle. The system initiated from several initial conditions and the destination was inside the landmark triangle. Figure 3 depicts the trajectories of the system. As we can see the controller was successful in navigating the system.

In the third navigation simulation, the landmarks were positioned on the vertices of an orthogonal triangle. The system initiated from several initial conditions and the destination was outside the landmark triangle. Figure 4 depicts

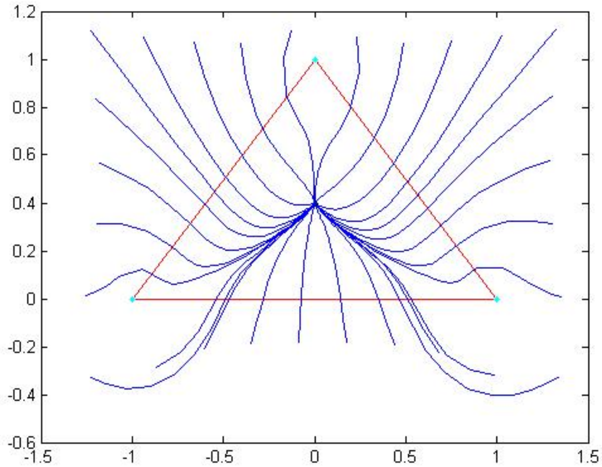

Fig. 3 *: Navigation:Flows of the system for various initial conditions with the destination inside the acute landmark triangle*

the trajectories of the system. As we can see the controller was successful and in this case in navigating the system.

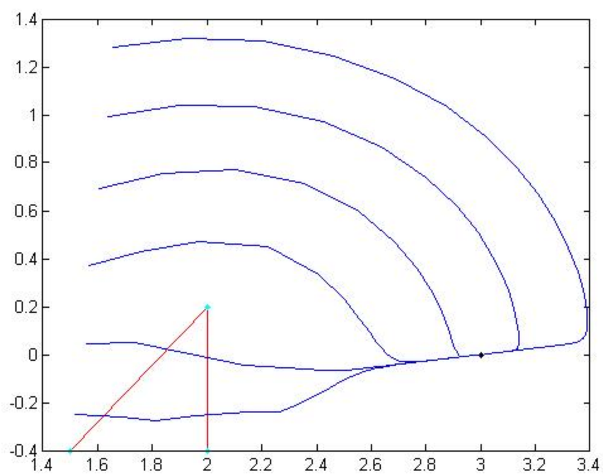

Fig. 4 *: Navigation:Flows of the system for various initial conditions with the destination outside an orthogonal landmark triangle*

B. Tracking Simulations

To verify the effectiveness of our tracking controller, we set up a simulation with non-stationary landmarks positioned on the vertices of an obtuse triangle. In the first simulation, the landmarks were rotating on same radius circles with the same frequencies. Figure 5 depicts the trajectories of the system and the landmarks.

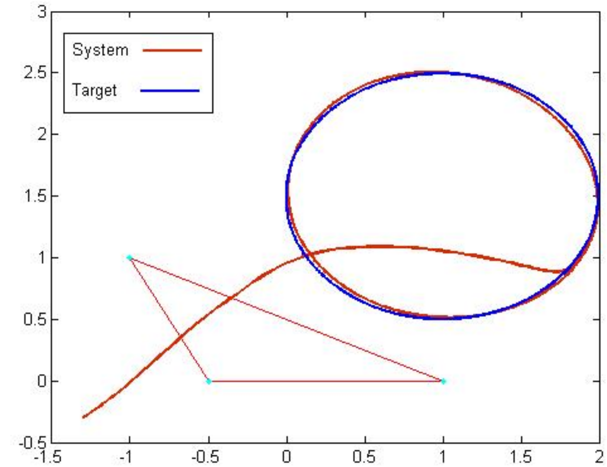

Fig. 5 *: Tracking:Trajectories of the system and the target*

The tracking error in cartesian coordinates is depicted in figure 6.

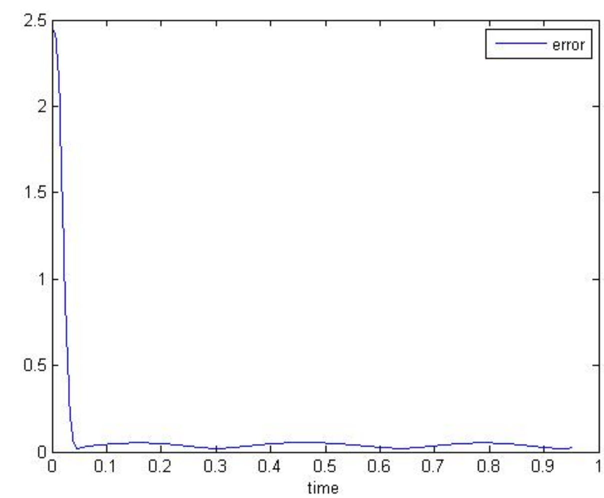

Fig. 6 *: Tracking:Euclidean tracking error*

As we can see the tracking error decreases and remains bounded throughout the task.

In figure 7 we can se the evolution of the system gain. As we can see the gain increases up to a maximum value as remains constant after that.

We see that the tracking controller performed as expected by containing the tracking error within a bound. The gain regulation controller successfully increased the gain up to the necessary amount such that the tracking controller could

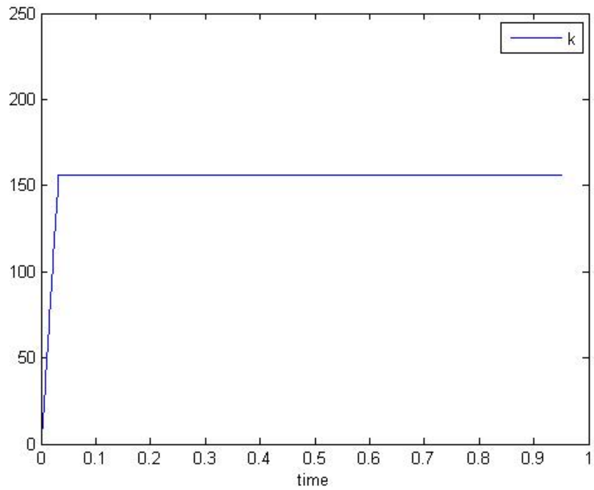

Fig. 7 *: Tracking: Controller gain*

achieve asymptotic tracking in the neighborhood of the target.

V. Conclusions

We have proposed a novel and provably correct technique for navigation and tracking by utilizing bearing only measurements from three landmarks of unknown location and geometry. This line of research was inspired by the capabilities of several ant species to navigate using only bearing information based on retinal positions of landmarks. The controllers that we have developed have analytically guaranteed global stability properties. Simulation results demonstrate the applicability of our techniques.

Further research includes extending this framework into more complicated systems with dynamics and nonholonomic constraints, and applying this techniques to swarm formation control type of scenarios. Other research directions includes investigating how the proposed bearing only control paradigm lends itself to constructing cooperative controllers for carrying out collective tasks in a distributed manner.

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