Stabilization of Multiple Robots on Stable Orbits via Local Sensing

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Abstract— **We develop decentralized controllers for a team of disk-shaped robots to converge to and circulate along the boundary of a desired two-dimensional geometric pattern specified by a smooth function with collision avoidance. The proposed feedback controllers rely solely on each robot's range and bearing sensors which allow them to obtain information about positions of neighbors within a given range. This is releveant for applications such as perimeter surveillance or containing hazardous regions where limited bandwidth must be preserved for situational awareness. The computational complexity of the decentralized controller for each agent is linear in the number of neighboring agents, making it scalable to robot swarms. We establish stability and convergence properties of the controllers and verify the feasibility of the method through computer simulations.**

I. Introduction

As the number of robots in a team increases, there has been a shift towards a swarming paradigm where robots are programmed with simple but identical behaviors that can be realized with limited on-board computational, communication and sensing resources. In nature, we often see such swarming behaviors, specifically in biological systems composed of large numbers of organisms which individually lack either the communication or computational capabilities required for centralized control. Some examples of such behaviors can be seen in the group dynamics in beehives [1], ant colonies [2], bird flocks, steer herds and fish schools [3].

We are interested in deploying robotic teams for perimeter surveillance and monitoring of specified areas, where robots may have to communicate with each other in order to integrate and fuse the information acquired by various sensors, and cooperative manipulation, where robots may need to surround an object to transport it from one location to another. Often times, limited bandwidth must be preserved to enable the communication of crucial data between team members and/or back to a base station. This is especially critical when we consider a swarming paradigm where bandwidth often becomes the limiting factor in agents' abilities to transmit data. In these situations, robots must not only have the ability to generate complex shapes in two dimensions and operate with little or no direct human supervision, they must also accomplish the task with as little communication overhead as possible.

Previous works in group coordination using decentralized controllers to synthesize geometric patterns include [4], which discussed decentralized control algorithms for line and circle formations. More general geometric patterns are considered in [5], however the approach requires every robot to have an estimate of the positions of all the other robots. In [6], Chaimowicz *et al.* presented a method for arbitrary shape generation using a swarm of robots using interpolated implicit functions. The stability and convergence properties of those controllers with inter-agent constraints was considered for a class of boundaries in [7]. A similar approach to distributed shape control using Fourier descriptors was considered in [8]. Pattern formation is achieved in [9] for a certain class of closed curves by determining each agent's distance with its neighbors and the desired contour. A similar problem was considered in [10] where it was formulated as global energy minimization task over the entire collective.

Approaches more suitable for applications like perimeter surveillance include [11] where the problem of detecting and tracking a specific environmental boundary is considered. Here the control laws are determined using a partial differential equation approach and require the communication of each agent's position to its nearest neighbors. In [12] and [13] the stabilization to isolated relative equilibria for a group of particles in the plane was achieved using certain types of communication interconnection topologies.

In our work, we build on the results of [7] and address the synthesis of decentralized controllers that guarantee the stability and convergence of all the robots to the boundary of a specified shape while maintaining a non-zero velocity allowing each robot to consistently travel along the desired boundary. We have developed an approach that does not require the robots to communicate with each other for these tasks. In our approach, collision avoidance is achieved via a prioritization schemed based on relative neighbors rather than gyroscopic forces as in [14]. While we assume that agents are holonomic, it is possible to extend our methodology to include non-holonomic robots. This can be achieved because we consider disk-shaped robots, thus enabling the use of feedback linearization techniques to linearize the non-holonomic model away from each robot's center of rotation.

This paper is divided into the following sections: In Section II, we formulate the problem and provide some background to our approach. Section III outlines our proposed control methodology. The properties of our controller, including safety and stability, are discussed in Section IV. Simulation results are presented in Section V. Finally, we discuss directions for future work in Section VI.

II. Problem Formulation

We consider a group of N planar, fully actuated robots each with kinematics given by

$$\dot{q}_i = u_i \tag{1}$$

where $q_i = (x_i, y_i)^T$ and u_i denote the i^{th} agent's position and control input. Thus, the robot state is a 2×1 state vector and the state of the team of robots is given by $\mathbf{q} = \left[q_1^T \dots q_N^T\right]^T \in \mathbf{Q} \subset \mathbb{R}^{2N}$. We assume each agent has a radius r_i.

We would like to design control inputs that will stabilize the group of N robots to the boundary (curve) of a desired smooth, compact set, e.g. shape, with non-zero tangential velocities to the boundary curve, enabling the agents to travel along the boundary curve in a counter-clockwise direction, all the while avoiding collisions with other agents. This is relevant for applications such as perimeter surveillance or cordoning off and containing hazardous regions after chemical spills or biological terrorist attacks.

We assume that the workspace, $\mathcal{W}$, is obstacle free and given by the set,

$$\mathcal{W} = \{q | \|q\| \leq R_0\}.$$

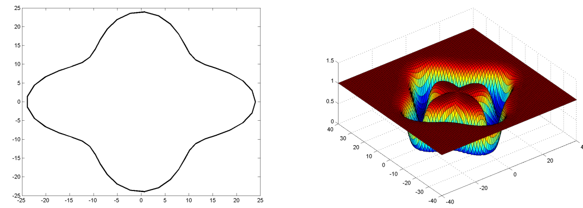
For a desired smooth star shape, $\mathcal{S}$, we denote $\partial\mathcal{S}$ to be the boundary of $\mathcal{S}$ and, similar to [7], we assume that $\partial\mathcal{S}$ is described by a smooth, regular, simple, closed curve $s(x, y) = 0$. In order to avoid inter-agent collisions, agents must have the ability to sense the proximity of their teammates. We define the neighborhood of q_i by the range and field of view of the sensing hardware and denote the set of neighbors in this region by Γ_i. Assuming a circular sensing range, denoted by R_i, collision avoidance maneuvers will occur when agents are within each other's sensing range and be determined by their relative distances.

The objective is to construct artificial potential functions, φ, and augment these with a rotational vector field to stabilize the team of N agents onto the desired closed curve (orbit) with non-zero velocities in the orbit's tangent space. We achieve collision avoidance by appropriately scaling the velocities that drives the individual agents towards and along the boundary curve. We outline our methodology in the following section.

III. Methodology

A. Assumptions

Given a smooth star shape $\mathcal{S}$, a team of N robots each with radius $r_i > 0$ and sensing range $R_i > 0$, we define $r = \max_i r_i$ and $R = \max_i R_i$. Our goal is to synthesize decentralized controllers that will allow a team to converge to the desired orbit with non-zero velocities in the orbit's tanget space while avoiding collisions. Therefore, the length of $\partial\mathcal{S}$, L, naturally imposes an upper bound on the number of robots, e.g. $N_{max} > 0$, that can travel along the boundary with non-zero velocity. Thus, we make the following assumptions:



(a) (b)

Fig. 1. (a) A star shaped shaped whose boundary is given by $r - (a + b\sin(c\theta + d)) = 0$ with $a = 20$, $b = 4$, $c = 4$, and $d = \pi/2$. (b) The shape navigation function for the boundary given in (a).

1) $N < N_{max}$;
2) $|\rho_{min}| > R$;
3) $\min_{s \in [\frac{\pi\rho_0}{2}, L - \frac{\pi\rho_0}{2}]} \|q_0(s) - q(s)\| > R$ for any $q_0(s) \in \partial\mathcal{S}$, where $s \in [0, L]$ denotes the arclength and ρ_0 denotes the radius of curvature at q_0.

Assumption 1 ensures agents will have non-zero velocities in the orbit's tangent space. Assumptions 2 and 3 ensure convergence by excluding boundaries with sharp turns and star shaped patterns with narrow corridors that may result in robots repelling each other away from the boundary while avoiding collisions. Furthermore, assumptions 2 and 3 ensure that the prioritization scheme for collision avoidance will be consistent throughout the workspace.

B. Controller Synthesis

Given a smooth star shape $\mathcal{S}$, we assume that $\partial\mathcal{S}$ is described by a smooth, regular, simple, closed curve $s(x, y) = 0$, with $s(x, y) < 0$ for all (x, y) in the interior of $\partial\mathcal{S}$ and $s(x, y) > 0$ for all (x, y) in the exterior of $\partial\mathcal{S}$. Let $\gamma = s(x, y)$ and $\beta_0 = R_0 - \|q\|^2$, we define the *shape navigation function*, φ, as

$$\varphi(q) = \frac{\gamma^2}{[\gamma^2 + \beta_0]}. \tag{2}$$

The function φ has the following properties:

- φ is positive semi-definite;
- $\varphi = 0$ if and only if $s(x, y) = 0$;
- φ is uniformly maximal, e.g. $\varphi(\partial\mathcal{W}) = 1$;
- φ is real analytic.

Figure 1 shows a star shaped boundary and its shape navigation function. The shape navigation function will generate an input that will drive each agent towards the desired boundary, $\partial\mathcal{S}$.

To enable the agents to travel along $\partial\mathcal{S}$ in a counter-clockwise direction, let $\psi = [0\,0\,\gamma]^T$ and we impose an additional input given by $-\nabla \times \psi$, where $\nabla \times \psi$ is a vector tangent to the level set curves of φ. Furthermore, $\nabla \times \psi$ is chosen such that on the boundary, $\partial\mathcal{S}$, each agent has a non-zero tangent velocity. This input enables each agent to travel along the boundary in a counter-clockwise direction[1].

[1]To enable each agent to travel along $\partial\mathcal{S}$ in a clockwise direction consider adding $(\nabla_i \times \psi)g(T_i)$ in (3) instead of subtracting.

Thus, the proposed decentralized controller is given by:

$$u_i = -\nabla_i \varphi_i \cdot f(N_i) - \nabla_i \times \psi_i \cdot g(T_i) \tag{3}$$

where ∇_i denotes differentiation with respect to agent i's coordinates and $\varphi_i = \varphi(q_i)$ and $\psi_i = \psi(q_i)$. The functions $f(N_i)$ and $g(T_i)$ are positive scalar functions used to modulate each agent's velocity for collision avoidance and their construction is described in the following section. The first term of (3) drives the agents towards $\partial\mathcal{S}$ and the second term drives the agents along the level set curves of φ in a counter-clockwise direction. Figures 2(a) and 2(b) show the vector fields given by the first term and second term of Equation (3) for a star shaped boundary with $f(N_i) = g(T_i) = 1$. The combined vector field described by Equation (3) is shown in Figure 2(b). In the remainder of this paper we will refer to the first and second term of (3) as the descent and tangential velocities of agent i.

C. Collision Avoidance

For collision avoidance, we want individual agents to have the ability to modulate their respective descent and tangential speeds based on their relative positions with respect to their neighbors. Consider the following scalar functions

$$N_{ij}(k) = \frac{(q_i - q_j)^T(-\nabla_j \varphi_j)}{(\|q_i - q_j\|^2 - (r_i + r_j)^2)^k} \tag{4}$$

$$T_{ij}(k) = \frac{(q_i - q_j)^T(-\nabla_j \times \psi_j)}{(\|q_i - q_j\|^2 - (r_i + r_j)^2)^k} \tag{5}$$

where k is a positive even number. As agent i descends towards $\partial\mathcal{S}$, for any $q_j \in \Gamma_i$ such that $N_{ij} > 0$, we will assign agent i priority over agent j. Similarly, as agent i travels along the level sets of φ, for any $q_j \in \Gamma_i$ such that $T_{ij} > 0$, agent i is given priority over agent j. Essentially, agents who are closer to the boundary and/or "traveling in front" of others will be given higher priorities than those farther from the boundary and/or "behind" others. To achieve this, we would like to construct the functions $f(N_i)$ and $g(T_i)$ in (3) such that q_i increases its descent and tangential velocities. Alternatively, if $N_{ij} < 0$ and/or $T_{ij} < 0$ for any $q_j \in \Gamma_i$, agent j would be assigned a higher priority and thus, we would like $f(N_i)$ and $g(T_i)$ in (3) to decrease the descent and tangential velocities of q_i accordingly.

Consider the following two real analytic switching functions,

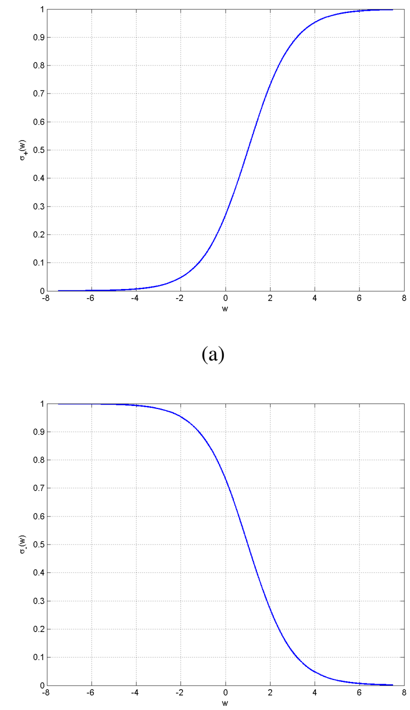
$$\sigma_+(w) = \frac{1}{1 + e^{1-w}},$$

$$\sigma_-(w) = \frac{1}{1 + e^{w-1}}.$$

We note that $\sigma_+ \to 0$ as $w \to -\infty$ and $\sigma_+ \to 1$ as $w \to +\infty$ and $\sigma_- \to 0$ as $w \to +\infty$ and $\sigma_- \to 1$ as $w \to -\infty$. Figure 3 shows the graphs of these two functions. We define the functions $f(N_i)$ and $g(T_i)$ as follows

$$f(N_i) = \sigma_+(N_i), \tag{6}$$

$$g(T_i) = 1 - \sigma_-(T_i), \tag{7}$$



(a)

(b)

Fig. 3. (a) Graph of $\sigma+(w)$. (b) Graph of $\sigma_-(w)$.

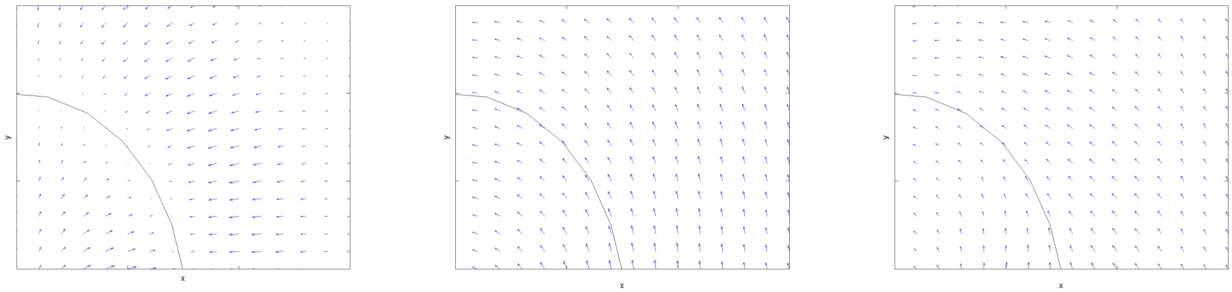
such that N_i and T_i are given by

$$N_i = \sum_{j\in\Gamma_i} \left(\frac{\sigma_+(N_{ij}(2))}{\|q_i - q_j\|^2 - (r_i + r_j)^2} - \frac{\sigma_-(N_{ij}(4))}{(\|q_i - q_j\|^2 - (r_i + r_j)^2)^2} \right), \tag{8}$$

$$T_i = \sum_{j\in\Gamma_i} \left(\frac{\sigma_+(T_{ij}(2))}{\|q_i - q_j\|^2 - (r_i + r_j)^2} - \frac{\sigma_-(T_{ij}(4))}{(\|q_i - q_j\|^2 - (r_i + r_j)^2)^2} \right). \tag{9}$$

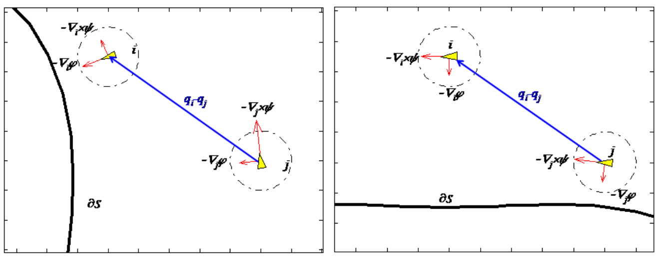
We have chosen N_i and T_i such that neighbors "in front" and "below" q_i have higher priority than those "behind" and "above" q_i. The functions $f(N_i)$ and $g(T_i)$ are constructed such that $f(N_i) \to 0$ and/or $g(T_i) \to 0$ as q_i approaches q_j from "above" and/or "behind". Figure 4 provides a schematic of the prioritization scheme. The boundary is denoted by the dark solid line and the size of the agents are shown by the dotted circles. The arrows correspond to the $-\nabla_i\varphi$ and $-\nabla \times \psi$ respectively. In the case where $N_{ij} > 0$, agent i has priority over agent j and similarly for $T_{ij} > 0$.

We note since the φ and ψ is common among all agents, robots do not have to exchange information. Instead, the positions of the neighbors can be obtained via sensing alone. Moreover, the computational complexity of the proposed controller for each agent is linear in $|\Gamma_i|$.



(a) (b) (c)

Fig. 2. Vector fields for the star shaped boundary given by $r - (a + b * sin(c\theta + d)) = 0$ with $a = 20$, $b = 10$, $c = 4$, and $d = \pi/2$. (a) Vector field given by $-\nabla\varphi$. (b) Vector field given by $-\nabla \times \psi$. (c) Vector field given by equation (3) with $f(N_i) = 1$ and $g(T_i) = 1$.



(a) $N_{ij} > 0$ (b) $N_{ij} < 0$

Fig. 4. The dark solid line denotes the boundary, the dotted circles denote the size of the agents, and the red arrows denote the descent and tangential velocities. (a) $N_{ij} > 0$, agent i has priority over agent j. (b) $N_{ij} < 0$, agent j has priority over agent i.

IV. Safety and Stability Results

In this section, we consider the stability and covergence properties of our controller given by (3) for a group of N robots each with kinematics given by (1). We present our results with the proofs located in the Appendix.

Our first two propositions concerns the safety of the system, e.g. no collisions can occur in finite time, and non-penetration between any two neighboring agents.

Proposition 4.1: Given $\partial\mathcal{S}$, the system of N robots with kinematics (1), the feedback control law (3) guarantees safety.

Corollary 4.2: Given $\partial\mathcal{S}$, the feedback control law (3) satisfies $(q_i - q_j)^T(v_i - v_j) \geq 0$ for all pairs of i,j where $\Gamma_i = \{i\}$ and $\Gamma_j = \{j\}$.

Remark 4.3: This result guarantees collision avoidance between the robots since we can either consider the disk-shaped robot sets to be open or we can assume an implied non-zero safety margin between the actual and modeled boundaries of the robots.

Our final proposition concerns the convergence of the system to the desired boundary where q_i^0 denotes the intial position of agent i.

Proposition 4.4: For any smooth star shape, $\mathcal{S}$, the system of N robots each with kinematics (1), control input (3), and $\|q_i^0\| > r$, the system converges asymptotically to $\partial\mathcal{S}$.

V. Simulations

We illustrate the proposed controller with some simulation results. Figure 5 shows a team of 20 robots each with radius of 2 converging towards a star shaped boundary. The desired boundary is denoted by the dotted line while the agents' trajectories are denoted by the solid lines. The circles denote the size of the agents and the triangles denote their headings. Similarly, Figure 6 shows a team of 30 robots each with radius of 1 converging to a pentagon-like boundary. We have intentionally limited the number of robots in these simulations in order to better display the individual robot trajectories.

VI. Conclusions and Future Work

We have presented an efficient decentralized approach for a team of robots to converge and track the boundary of a desired two-dimensional shape while avoiding collisions. The algorithm can be used to deploy multiple robots to do perimeter surveillance or to cordon off hazardous areas. The algorithm is scalable to large number of robots since control inputs solely rely on information obtained from each robot's sensors thus preserving bandwidth for critical data transfers. Additionally, the computational complexity of the decentralized controller for each agent is linear in the number of neighboring agents. The controller was shown to be stable and convergence to the boundary of star shaped sets was established. Moreover, the methodology ensures that collision avoidance is achieved between the robots.

There are many directions for future work. We would like to further investigate additional topological requirements on the level set curves of our shape navigation functions to enable the extension of our results to more general two-dimensional patterns. We would also like to extend our methodologies to enable tracking of time-varying boundaries. Lastly, we would like to extend our methodology such that it would be robust to imperfect sensing and failure of individual agents.

Acknowledgements

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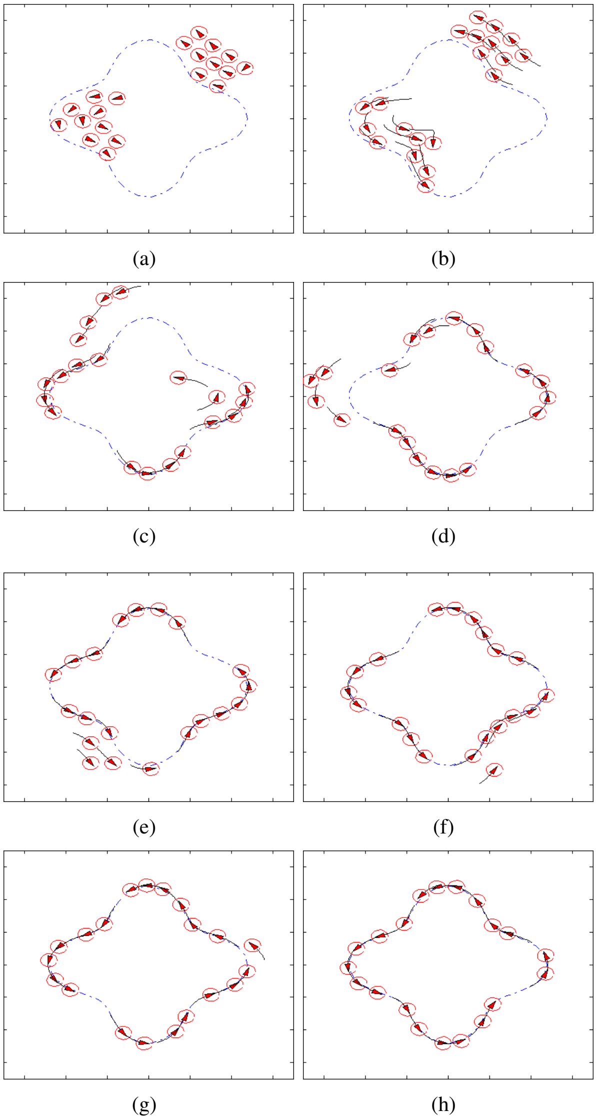

Fig. 5. A team of 20 robots each with radius of 2 and converging towards a star shaped boundary. The red circles denotes the size of the robots and the red triangles denotes the heading. The dotted line denotes the desired boundary and the solid lines denote the agents' trajectories.

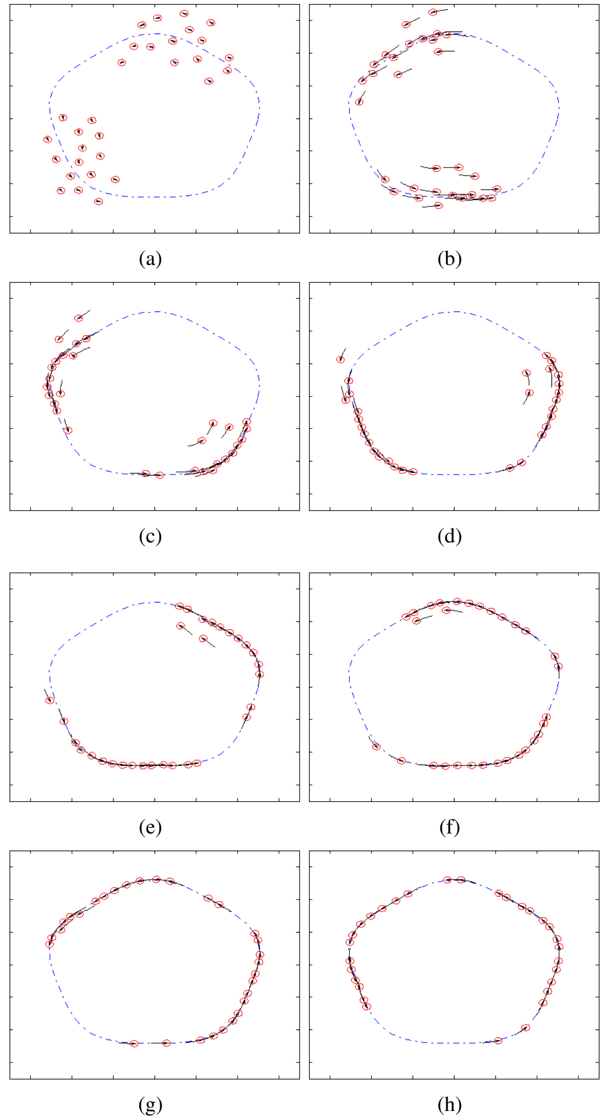

Fig. 6. A team of 30 robots each with radius of 1 and converging towards a pentagon-like boundary. The red circles denotes the size of the robots and the red triangles denotes the heading. The dotted line denotes the desired boundary and the solid lines denote the agents' trajectories.

APPENDIX

Proof for Proposition 4.1: For any i,j with $\|q_i - q_j\| < \min\{R_i, R_j\}$, such that $i \in \Gamma_j$ and $j \in \Gamma_i$ (a minimum distance is guaranteed to exist since R_i,$R_j > 0$). Consider the following:

$$\begin{aligned}(q_i - q_j)^T(v_i - v_j) \\ &= (q_i - q_j)^T \left[(-\nabla_i \varphi_i f(N_i) - (\nabla_i \times \psi_i) g(T_i))\right. \\ &- \left.(-\nabla_j \varphi_j f(N_j) - (\nabla_j \times \psi_j) g(T_j))\right] \\ &= (q_j - q_i)^T (\nabla_i \varphi_i f(N_i) + (\nabla_i \times \psi_i) g(T_i)) \\ &+ (q_i - q_j)^T (\nabla_j \varphi_j f(N_j) + (\nabla_j \times \psi_j) g(T_j)) \\ &= -(\|q_i - q_j\|^k - (r_i + r_j)^k) \left[N_{ji} f(N_i)\right. \\ &+ \left.T_{ji} g(T_i) + N_{ij} f(N_j) + T_{ij} g(T_j)\right] \qquad (10)\end{aligned}$$

As $\|q_i - q_j\| \to (r_i + r_j)$, $(q_i - q_j)^T(v_i - v_j) \to 0$. ■

Proof for Corollary 4.2: Consider the following four cases for Equation (10):

a) Case I: $N_{ij} \leq 0$ and $T_{ij} \leq 0$: By construction we can conclude $N_{ji} \geq 0$ and $T_{ji} \geq 0$. Furthermore, $0 \leq f(N_i) \leq f(N_j)$ and $0 \leq g(T_i) \leq g(T_j)$.

b) Case II: $N_{ij} \geq 0$ and $T_{ij} \geq 0$: This implies $N_{ji} \leq 0$ and $T_{ji} \leq 0$ and thus $0 \leq f(N_j) \leq f(N_i)$ and $0 \leq g(T_j) \leq g(T_i)$.

c) Case III: $N_{ij} \leq 0$ and $T_{ij} \geq 0$: This results in $N_{ji} \geq 0$ and $T_{ji} \leq 0$ and therefore, $0 \leq f(N_i) \leq f(N_j)$ and $0 \leq g(T_j) \leq g(T_i)$.

d) Case IV: $N_{ij} \geq 0$ and $T_{ij} \leq 0$: Then $N_{ji} \leq 0$ and $T_{ji} \geq 0$ with $0 \leq f(N_j) \leq f(N_i)$ and $0 \leq g(T_i) \leq g(T_j)$. Therefore, for all four cases, the expression

$[N_{ji}f(N_i) + T_{ji}g(T_i) + N_{ij}f(N_j) + T_{ij}g(T_j)] \leq 0$ thus $(q_i - q_j)^T(v_i - v_j) \geq 0$. ■

Proof for Proposition 4.4: To show that the system converges asymptotically to $\partial\mathcal{S}$, we first consider the stability of the proposed controller. Consider the following positive semi-definite function:

$$V(\mathbf{q}) = \sum_i \varphi(q_i). \tag{11}$$

The set $\mathcal{W}$ is compact and since $\varphi_i \leq 1$ for all i and thus the level sets of φ are compact subsets of $\mathcal{W}$. Then the time derivative of V is given by

$$\begin{aligned}\dot{V} &= \sum_i \nabla\varphi(q_i)^T\dot{q}_i \\ &= \sum_i \nabla_i\varphi(q_i)^T\left(-\nabla_i\varphi_i f(N_i) - (\nabla_i \times \psi)g(T_i)\right)\end{aligned}$$

By construction, $\nabla\varphi$ is orthogonal to $\nabla \times \psi$, therefore the above equation simplifies to

$$\dot{V} = \sum_i -\|\nabla_i\varphi_i\|^2 f_i(N_i) \leq 0 \tag{12}$$

since $f(\cdot) \in [0, 1]$. By LaSalle's Invariance Principle, for any initial condition in $\mathcal{W}$, the system of N agents with kinematics (1), approaches the largest invariant set $\Omega_I = \{\mathbf{q} \in Q | \dot{V}(\mathbf{q}) = 0\}$. This shows that the proposed controller, given by Equation (3) is stable.

To show convergence, we first show that the closed loop system does not admit stationary points, i.e. points where $\dot{\mathbf{q}} = 0$ as long as $\|q_i^0\| > r$. Given $N < N_{max}$, we begin by considering the situation where $\gamma(q_i) > 0$ for all i, e.g. all N robots are outside of $\partial\mathcal{S}$. For $u_i = 0$ for all i, each robot must have zero descent and zero tangential velocities. By construction, for every robot to have zero tangential velocity, the robots must form a closed chain enclosing $\partial\mathcal{S}$ such that every robot is touching at least one other robot. By construction, for every robot to have zero descent velocity, there must exist a q_j such that $\varphi_j < \varphi_i$ for every i. Let $q_{min} = \arg\min_i \varphi(q_i)$, then by definition q_{min} cannot have a neighbor q_k such that $\varphi_k < \varphi(q_{min})$. Furthermore, since $N < N_{max}$, there are not enough robots to form a closed chain enclosing $\partial\mathcal{S}$. Thus, we can conclude $\exists l \in \{1, \ldots, N\}$ such that $u_l \neq 0$ if $\gamma(q_i) > 0$ for all i.

Similarly, consider the situation when $\gamma(q_i) < 0$ for all i, e.g. all N robots are inside $\partial\mathcal{S}$. For $u_i = 0$ for all i, each robot must have zero descent and tangential velocities. For robots to each have zero tangential velocity, the N robots must form a closed chain enclosing the set $\|q\| \leq r$. For robots to each have zero descent velocity, there must exist a q_j for every q_i such that $\varphi_j < \varphi_i$. Let $q_{min} = \arg\min_i \varphi(q_i)$, then by definition, q_{min} cannot have a neighbor q_k such that $\varphi_k < \varphi(q_{min})$. Thus, we can conclude, $\exists l \in \{1, \ldots, N\}$ such that q_l has a non-zero descent velocity. Therefore, if there is such a configuration where $u_i = 0$ for all i, it must be a configuration where some agents are inside and some are outside the boundary.

Without loss of generality we begin with two robots, q_a and q_b such that $\gamma(q_a) < \gamma(q_b)$ and $\gamma(q_a) \neq 0$, $\gamma(q_b) \neq 0$. Assume q_b is positioned such that $u_b = 0$. Then since $\gamma(q_a) < \gamma(q_b)$, by construction $\|q_a - q_b\| = (r_a + r_b)$ with $f(N_a) = f(N_b) = 0$. However, since both robots are traveling counter-clockwise along the their respective level set curves, q_a must have a non-zero tangential velocity. Thus, for $u_a = 0$, $\exists q_c$ such that $\gamma(q_c) < \gamma(q_a)$. Following this method of reasoning, for $u_i = 0\ \forall i$, either $\|q_i\| \leq r$ for some i in the set $\{1, \ldots, N\}$, or $\min_{s\in[\frac{\pi\rho_0}{2}, L-\frac{\pi\rho_0}{2}]}\|q_0(s) - q(s)\| \leq R$ for some $q_0 \in \partial\mathcal{S}$, or N must diverge, e.g. there are infinitely many robots. However, since $N < N_{max}$, N cannot diverge, $\|q_i\| > r$ for all i by Equation (12), and since $\min_{s\in[\frac{\pi\rho_0}{2}, L-\frac{\pi\rho_0}{2}]}\|q_0(s) - q(s)\| > R$ for all $q_0 \in \partial\mathcal{S}$, there can be no q_i such that $\|q_i\| \leq r$. Therefore, there must exist a q_j such that $u_j \neq 0$.

Finally to show asymptotic convergence to $\partial\mathcal{S}$, since there can be no stationary points, for initial conditions $\|q_i^0\| > r_i$, we conclude $\dot{V} = 0$ in (12) if and only if $\nabla_i\varphi_i = 0$ for all i. For a star shape, with φ given by (2), $\nabla\varphi = 0$ if and only if $q \in \partial\mathcal{S} \in \mathcal{W}$. Thus, the system of N robots converges asymptotically to $\partial\mathcal{S}$. ■

References

[1] N. F. Britton, N. R. Franks, S. C. Pratt, and T. D. Seeley, "Deciding on a new home: how do honeybees agree?" *Proc R Soc Lond B*, vol. 269, 2002.

[2] S. C. Pratt, "Quorum sensing by encounter rates in the ant temnothorax albipennis," *Behavioral Ecology*, vol. 16, no. 2, 2005.

[3] J. K. Parrish, S. V. Viscido, and D. Gr"unbaum, "Self-organized fish schools: An examination of emergent properties," *Biol. Bull.*, vol. 202, June 2002.

[4] O. Albayrak, "Line and circle formation of distributed autonomous mobile robots with limited sensor range," Ph.D. dissertation, Naval Postgraduate School, Monterey, CA, June 1996.

[5] I. Suzuki and M. Yamashita, "Distributed anonymous mobile robots: Formation of geometric patterns," *SIAM Journal on Computing*, 1999.

[6] L. Chaimowicz, N. Michael, and V. Kumar, "Controlling swarms of robots using interpolated implicit functions," in *Proceedings of the 2005 International Conference on Robotics and Automation (ICRA05)*, Barcelona, Spain, 2005.

[7] M. A. Hsieh and V. Kumar, "Pattern generation with multiple robots," in *International Conference on Robotics and Automation (ICRA) 2006*, Orlando, FL, April 2006.

[8] S. Kalantar and U. R. Zimmer, "Distributed shape control of homogeneous swarms of autonomous underwater vehicles," *to appear in Autonomous Robots (intl. journal)*, 2006.

[9] F. Zhang and N. E. Leonard, "Coordinated patterns of unit speed particles on a closed curve," *Systems and Control Letters (submitted)*, 2006.

[10] M. Jenkin and G. Dudek, "The paparazzi problem," in *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) 2000*, Takamatsu, Japan, October 2000, pp. 2042–2047.

[11] A. Bertozzi, M. Kemp, and D. Marthaler, "Determining environmental boundaries: Asynchronous communication and physical scales," in *Cooperative Control*, 2004.

[12] R. Sepulchre, D. Paley, and N. E. Leonard, "Stabilization of planar collective motion, part i: All-to-all communication," *Accepted for publication in IEEE Transactions on Automatic Control*, 2005.

[13] ——, "Stabilization of planar collective motion with limited communication," *Submitted to IEEE Transactions on Automatic Control*, 2006.

[14] D. E. Chang, S. Shadden, J. E. Marsden, and R. Olfati-Saber, "Collision avoidance for multiple agent systems," in *Proc. IEEE Conference on Decision and Control*, Maui, Hawaii, 2003.