CENTRALIZED FEEDBACK STABILIZATION OF MULTIPLE NONHOLONOMIC AGENTS UNDER INPUT CONSTRAINTS [1]

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Abstract: In this paper we use the centralized multirobot navigation function methodology established by the authors, augmented with an enhanced dipolar navigation field suitable for non-holonomic vehicles. A properly designed discontinuous feedback control law is applied to steer the nonholonomic vehicles. The resulting high frequency switching behavior, known as chattering, which is inherent in discontinuous controllers, is tackled with an appropriately designed discontinuous backstepping controller, which effectively reduces chattering. The vehicles are assumed to have limitations on their maximum achievable velocities. Those issues are taken into account in the control law design. The resulting closed form control scheme provides robust navigation with guaranteed collision avoidance and global convergence properties, as well as fast feedback, rendering the methodology particularly suitable for real time implementation, on systems with limited actuation capabilities. Collision avoidance and global convergence properties are verified through non - trivial computer simulations.

1. INTRODUCTION

Multiple robot navigation is a research area with an increasing research interest over the last decade. In the last few years multi - robot navigation for Non - Holonomic vehicles is gaining increasing attention.

Our main interest is to deduce global convergent control schemes with collision avoidance, suitable for real time implementation. In (Svestka and Overmars, 1995) a global convergent algorithm is presented for nonholonomic path planning, based on probabilistic roadmaps, but the me thodology cannot be used for real time implementation due to its complexity.

Nonholonomic stabilization has attracted the attention of the control community over the years, due to the fact that nonholonomic systems do not satisfy the Brockett's necessary smooth feedback stabilization condition (Brockett, 1981). In this paper we address the problem of nonholonomic navigation of multiple robots whose maximum translational and rotational velocities are upper bounded. To this extend, we use the concept of multi-robot navigation functions established by the authors (Loizou and Kyriakopoulos, 2002*a*), augmented with a dipolar (Tanner and Kyriakopoulos, 2000; Tanner *et al.*, 2003; Loizou and Kyriakopoulos, 2003) structure. The dipolar structure introduced in this paper, provides better non-holonomic navigation results, in terms of reduced chattering behavior, especially close to the target. A discontinuous backstepping controller, appropriately designed, acts as a low pass

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filter, smoothing out the chattering behavior. The resulting control law respects the velocity constraints, while maintaining the global convergence and collision avoidance properties.

The rest of the paper is organized as follows: Section 2 introduces the motivating problem. Section 3 outlines the concept of multi-robot dipolar navigation functions and introduces a new dipolar construction. Section 4 presents the nonholonomic control scheme. Section 5 presents the discontinuous backstepping controller design, section 6 presents simulation results for a number of non - trivial navigational tasks. Finally, section 7 summarizes the conclusions and indicates our current research directions.

2. PROBLEM STATEMENT

Consider the following system of m nonholonomic vehicles:

$$\dot{\mathbf{x}} = g\left(\mathbf{x}\right) \cdot \xi\left(\mathbf{x}\right) \tag{1}$$

where

$$\begin{aligned}
&\mathbf{x} = \left[\mathbf{x}_1^T \ldots \mathbf{x}_m^T\right]^T \in \left\{R^2 \times (-\pi, \pi]\right\}^m \\
&\mathbf{x}_i = \left[\, x_i \;\; y_i \;\; \theta_i \,\right]^T \in R^2 \times (-\pi, \pi] \\
&g\left(\mathbf{x}\right) = \left[g_1\left(\mathbf{x}_1\right)^T \ldots g_m\left(\mathbf{x}_m\right)^T\right]^T \in R^{3m \times 2} \\
&\xi = \left[\xi_1\left(\mathbf{x}\right)^T \ldots \xi_m\left(\mathbf{x}\right)^T\right]^T \in R^{2m} \\
&g_i\left(\mathbf{x}_i\right) = \begin{bmatrix} u_{\max}\cos\left(\theta_i\right) & 0 \\ u_{\max}\sin\left(\theta_i\right) & 0 \\ 0 & w_{\max} \end{bmatrix} \in R^{3 \times 2} \\
&\xi_i\left(\mathbf{x}\right) = \begin{bmatrix} u_i\left(\mathbf{x}\right) \\ w_i\left(\mathbf{x}\right) \end{bmatrix} \in R^2
\end{aligned}$$

with $i \in \{1 \ldots m\}$ and w_{max} and u_{max} are positive parameters representing the maximum rotational and translational velocity capabilities of the robots. (x_i, y_i, θ_i) are the position and orientation of each robot and u_i and w_i are the translational and rotational velocity controls. The input constraints are:

$$\begin{aligned} |u_i| \leq 1 \\ |w_i| \leq 1 \end{aligned} \tag{2}$$

The problem can be now stated as follows: *"Given the nonholonomic system (1), and the input constraints (2), derive a feedback kinematic control law that steers the system from any initial configuration to the goal configuration avoiding collisions. The environment is assumed perfectly known and stationary, while each robot acts as a potential obstacle to the others."*

3. MULTI-ROBOT DIPOLAR NAVIGATION FUNCTIONS

In previous works, the authors presented an extension to the navigation function methodology with applications to multiple robot navigation (Loizou and Kyriakopoulos, 2002*a*), and an extension of the methodology to multiple non-holonomic vehicles (Loizou and Kyriakopoulos, 2003), based on the dipolar field construction proposed in (Tanner and Kyriakopoulos, 2000). In this section we introduce a dipolar field construction, which results in smoother convergence to the target for the nonholonomic vehicle.

As it was shown in (Loizou and Kyriakopoulos, 2002*a*) the function: $\varphi = \frac{\gamma_d}{\left(\gamma_d^k + G\right)^{1/k}}$ proposed by (Koditschek and Rimon, 1990) for single robot navigation, with a proper selection of G can be used for multiple robot navigation and can be made a navigation function by an appropriate choice of k. Our assumption that we have spherical robots and spherical obstacles does not constrain the generality of this work since it has been proven (Koditschek and Rimon, 1990) that navigation properties are invariant under diffeomorphisms. Methods for constructing analytic diffeomorphisms are discussed in (Rimon and Koditschek, 1992) for point robots and in (Tanner *et al.*, 2003)for rigid body robots.

Let us assume the following situation: We have m mobile robots, and their workspace $W \subset R^2$. Each robot R_i, $i = 1 \ldots m$ occupies a disk in the workspace: $R_i = \left\{q \in R^2 : \|q - p_i\| \leq r_i\right\}$ where $p_i = \left[\, x_i \;\; y_i \,\right]^T$ is robot position and r_i is the radius of the robot. The configuration space C is spanned by $\mathbf{x}$. The destination configurations are denoted by the subscript d.

To be able to produce a dipolar potential field, φ must be modified as follows:

$$\varphi = \frac{\gamma_d}{\left(\gamma_d^k + H_{nh} \cdot G\right)^{1/k}} \tag{3}$$

where H_{nh} has the form of a pseudo - obstacle. Selecting H_{nh} as

$$H_{nh} = \varepsilon_{nh} + \left(\prod_{i=1}^{m} \eta_{nh_i}\right)^{\mu}$$

, guarantees that the navigation properties are not affected (Loizou and Kyriakopoulos, 2002*b*), as long as the workspace is bounded, η_{nh_i} can be bounded in the workspace and $\varepsilon_{nh} > \varepsilon\left(k\right)$. The choice we propose in this paper for η_{nh_i} is:

$$\eta_{nh_i} = \frac{\sin\left(\frac{1}{2}\pi\frac{\delta_i^2}{1+\delta_i^2}\right)}{\frac{1}{2}\pi\frac{\rho_i^2}{1+\rho_i^2}} \tag{4}$$

where $\rho_i = \|p_i - p_{d_i}\|$, $p = \left[p_1^T \ldots p_m^T\right]^T$, $\delta_i = \|(p - p_d) \cdot \mathbf{n}_{d_i}\|$, with

$$\mathbf{n}_{d_i} = \left[O_{1x2(i-1)} \;\; \cos\left(\theta_{d_i}\right) \;\; \sin\left(\theta_{d_i}\right) \;\; O_{1x2(m-i)}\right]^T$$

. μ is a positive tuning parameter and $\gamma_d = \|p - p_d\|^2 + \sum\limits_{j=1}^{m} k_\theta \Delta_\theta\left(\theta_\iota, \theta_{d_i}\right)^2$, where $k_\theta > 0$ and

$\Delta_\theta(\theta_1, \theta_2) = \arg\left(e^{i(\theta_1 - \theta_2)}\right)$ is a function that returns the angular difference in the range $(-\pi, \pi]$.

The intuition behind the choice of eq. (4) is that robot positions that rely on the axis passing through the target and are oriented along the robot's destination angle, are more favored, i.e. incur a lower cost to the potential function. Taking the limit of eq. (4) as the robot approaches it's destination, we get: $\lim_{\rho_i \to 0} \eta_{nh_i} = \cos(\tau)^2$, where τ is the relative angle between the robot approaching direction at the destination and the destination orientation. If we consider the convergence procedure, where the potential function is continuously decreasing, this implies that this limit is continuously increasing until its maximum which is for $\tau = 0$, i.e. the trajectories tends to eventually lay on the axis that passes through the destination and is oriented along the destination angle.

4. NON - HOLONOMIC CONTROL

In the following analysis we will use $V = \varphi$ as a Lyapunov function candidate, for notational consistency.

Define $M = \{1, \ldots, m\}$ and $\Omega = P(M)$ where P denotes the power set operator. Assuming that Ω is an ordered set, let N_j denote the j 'th element of Ω where $j \in \{1, \ldots, 2^m\}$. Then $N_j \subseteq M$ with $N_1 = \{\emptyset\}$ and $N_{2^m} = M$. We can now define: $\Delta_j = \frac{w_{max}}{\pi} \sum\limits_{i \in \{M \setminus N_j\}} (V_{\theta_i} \cdot \Delta_\theta(\theta_{nh_i}, \theta_i)) - u_{max} \sum\limits_{i=1}^{m} \left(|V_{x_i} \cdot \cos(\theta_i) + V_{y_i} \cdot \sin(\theta_i)| \cdot \frac{Z_i}{a_1 + Z_i} \right) - w_{max} \sum\limits_{i \in N_j} \frac{V_{\theta_i}^2}{a_2 + |V_{\theta_i}|}$ with $Z_i = a_3 \cdot \left(V_{x_i}^2 + V_{y_i}^2\right) + a_4 \left((x_i - x_{d_i})^2 + (y_i - y_{d_i})^2 \right)$ and

$$\theta_{nh_i} = atan2(V_{y_i} \cdot side_i, V_{x_i} \cdot side_i)$$

with

$$side_i = sgn((p - p_d) \cdot \mathbf{n}_{d_i})$$

and V_x, V_y, V_θ denotes the derivative of V along x, y, θ respectively. a_1, a_2, a_3, a_4 are positive constants. Define $H = \{j : \Delta_j < 0\}$ and $\rho = \left\{ j : \Delta_j = \max\limits_{i \in H} (\Delta_i) \right\}$. We can now state the following:

Proposition 1. The system (1) under the control law:

$$\omega_i = \frac{1}{\pi} \cdot \Delta_\theta(\theta_{nh_i}, \theta_i), \ i \in M \ \ \Delta_1 \leq 0$$
$$\omega_l = \frac{1}{\pi} \cdot \Delta_\theta(\theta_{nh_l}, \theta_l), \ l \in \{N_\rho\}, \ \Delta_1 > 0$$
$$\omega_j = -\frac{V_{\theta_j}}{a_2 + |V_{\theta_i}|}, \ j \in \{M \setminus N_\rho\}, \ \Delta_1 > 0$$

$$u_i = -\text{sgn}(V_{x_i} \cdot \cos(\theta_i) + V_{y_i} \cdot \sin(\theta_i)) \cdot \frac{Z_i}{a_1 + Z_i}, \quad i \in M$$

is globally asymptotically stable.

PROOF. The navigation function V studied in the previous section serves as a Lyapunov function candidate. We will now examine the derivative of V along the trajectories of (1): $\dot{V} = \frac{\partial V}{\partial t} + \nabla V \cdot \dot{\mathbf{x}} = \nabla V \cdot g(\mathbf{x}) \cdot \xi(\mathbf{x})$ since $V = V(\mathbf{x})$ with $\nabla V = \left[\frac{\partial V}{\partial x_1} \ \frac{\partial V}{\partial y_1} \ \frac{\partial V}{\partial \theta_1} \ \cdots \ \frac{\partial V}{\partial x_m} \ \frac{\partial V}{\partial y_m} \ \frac{\partial V}{\partial \theta_m} \right]^T$. Substituting we get:

$$\dot{V} = \sum_{i=1}^{m} \big(u_i \cdot u_{\max} \left(V_{x_i} \cos(\theta_i) + V_{y_i} \sin(\theta_i) \right) + w_i \cdot w_{\max} \cdot V_{\theta_i} \big)$$

We are interested in establishing that $\dot{V} < 0$ almost everywhere, and the sets of points where $\dot{V} = 0$ except from the destination are not invariant. Applying the proposed controls, we get:

For $\Delta_1 \leq 0$ we have:

$$w_i = \frac{1}{\pi} \cdot \Delta_\theta(\theta_{nh_i}, \theta_i), \ i \in M$$

$$u_i = -\text{sgn}(V_{x_i} \cdot \cos(\theta_i) + V_{y_i} \cdot \sin(\theta_i)) \cdot \frac{Z_i}{a_1 + Z_i}, \quad i \in M$$

Then $\dot{V} = \Delta_1 \leq 0$. To proceed with the proof we will need the following lemma:

Lemma 1. If $\Delta_1 > 0$ then $\exists i \in \{1, \ldots, 2^m\} : \Delta_i < 0$

PROOF. If $\Delta_1 > 0$ then since:

$$-K_u \sum_{i=1}^{m} \left(|V_{x_i} \cdot \cos(\theta_i) + V_{y_i} \cdot \sin(\theta_i)| \cdot Z_i \right) \leq 0$$

It must be $\frac{w_{max}}{\pi} V_{\theta_i} \cdot \Delta_\theta(\theta_{nh_i}, \theta_i) > 0$, which means that there exists at least one k for which $V_{\theta_k} \neq 0$ and the term $-K_\theta \cdot \sum\limits_{i \in N_j} V_{\theta_i}^2$ of some Δ_i will be negative definite. For the worst case scenario, $\Delta_{2^m} < 0$ since $N_{2^m} = M$. $\square$

For $\Delta_1 > 0$ then there is at least one j for which $\Delta_j < 0$ as we deduced from (Lemma 1) and thus $\rho \neq \{\emptyset\}$. We choose $j = \rho$ because we want the maximum possible number of robots to follow the dipole generated Non-Holonomic trajectories. The rest will be doing a conflict avoidance manoeuver. The controls in those cases take the form:

$$\omega_l = \frac{1}{\pi} \cdot \Delta_\theta(\theta_{nh_l}, \theta_l), \ l \in \{N_\rho\}, \ \Delta_1 > 0$$
$$\omega_j = -\frac{V_{\theta_j}}{a_2 + |V_{\theta_i}|}, \ j \in \{M \setminus N_\rho\}, \ \Delta_1 > 0$$

$$u_i = -\mathrm{sgn}\left(V_{x_i} \cdot \cos\left(\theta_i\right) + V_{y_i} \cdot \sin\left(\theta_i\right)\right) \cdot \frac{Z_i}{a_1 + Z_i}, \quad i \in M$$

Then $\dot{V} = \Delta_\rho \leq 0$

Now let $E = \{\mathbf{x} : \dot{V}(\mathbf{x}) = 0\}$ and $E \supset S = \{x : \omega_i = u_i = 0, \forall i \in M\}$ is an invariant set. From the proposed control law, it can be seen that $u_i = 0, \forall i \in M$ only at the destination, and for all other configurations the controller provides a direction of movement. According to LaSalle's invariance principle, the trajectories of the system converge asymptotically to the largest invariant set, which is the destination configuration □

Proposition 2. The control law defined in Proposition 1 respects the input constraints defined in (2).

PROOF. Taking the absolute values, we get:

$$\begin{aligned}
|u_i| &= \frac{Z_i}{a_1 + Z_i} \leq 1, \quad i \in M \\
|w_i| &= \frac{1}{\pi}\left|\Delta_\theta\left(\theta_{nh_i}, \theta_i\right)\right| \leq 1, \quad i \in M, \Delta_1 \leq 0 \\
|w_l| &= \frac{1}{\pi}\left|\Delta_\theta\left(\theta_{nh_l}, \theta_l\right)\right| \leq 1, \quad l \in \{N_\rho\}, \Delta_1 > 0
\end{aligned}$$

since $\Delta_\theta\left(\theta_{nh_i}, \theta_i\right) \in (-\pi, \pi]$, and

$$|w_j| = \frac{\left|V_{\theta_j}\right|}{a_2 + \left|V_{\theta_j}\right|} \leq 1, \quad j \in \{M \backslash N_\rho\}, \Delta_1 > 0$$

□

5. DISCONTINUOUS BACKSTEPPING CONTROLLER

Inherent in discontinuous controllers is the fast switching behavior which occurs over the sliding surfaces. According to Filippov, the solutions over the sliding surfaces are absolutely continuous and flow tangentially to the sliding surface. Unfortunately, due to the discrete time integration of the control signal, a behavior known as chattering emerges when it comes to discontinuous controllers. We tackle this issue by implementing a discontinuous backstepping controller, which acts as a low pass filter, smoothing out the system's behavior over the sliding surfaces. Consider the initial system (1) augmented with virtual states, as follows:

$$\begin{aligned}
\dot{\mathbf{x}} &= g(\mathbf{x}) \cdot \xi'(\mathbf{x}, \mathbf{z}) \\
\dot{\mathbf{z}} &= -c \cdot \mathbf{z} - \beta(\mathbf{x}, \mathbf{z})
\end{aligned} \tag{5}$$

Define $M_b = \{1, \ldots, m\}$ and $\Omega_b = P(M_b)$ where P denotes the power set operator. Assuming that Ω_b is an ordered set, let ${}^bN_j$ denote the j 'th element of Ω_b where $j \in \{1, \ldots, 2^m\}$. Then ${}^bN_j \subseteq M_b$ with ${}^bN_1 = \{\emptyset\}$ and ${}^bN_{2^m} = M_b$. We can now define: ${}^b\Delta_j = A + \sum_{i \in \{M_b \backslash {}^bN_j\}} k_2 z_{2_i} \frac{w_{\max}}{\pi} \Delta_\theta\left(\theta_{nh_i}, \theta_i\right) - \sum_{i \in {}^bN_j} w_{\max} k_2 z_{2_i} b_i \cdot V_{\theta_i}$, where

$$\begin{aligned}
A = \sum_{i=1}^{m} & b_i \left(u_{\max} \cdot u_i \left(V_{x_i} \cos\theta_i + V_{y_i} \sin\theta_i\right)\right. \\
& \left. + w_{\max} \cdot w_i \cdot V_{\theta_i}\right) \\
& + \sum_{i=1}^{m} \left(b_i w_{\max} k_2 z_{2_i} V_{\theta_i} - k_2 c_2 z_{2_i}^2 - k_1 c_1 z_{1_i}^2\right)
\end{aligned}$$

, and

$$b_i = \left(1 + |k_1 z_{1_i}| + |k_2 z_{2_i}| + |u_i| + |w_i|\right)^{-1}$$

Define $H_b = \left\{j : {}^b\Delta_j < 0\right\}$ and

$$\rho_b = \left\{j : {}^b\Delta_j = \max_{i \in H_b}\left({}^b\Delta_i\right)\right\}$$

. We can now state the following:

Proposition 3. If u and w are stabilizing controllers of the system (1) and V a Lyapunov function, then system (5) under the control law:

$$\begin{aligned}
\xi_i' &= b_i \cdot \begin{bmatrix} \left(k_1 z_{1_i} + u_i\right) \\ \left(k_2 z_{2_i} + w_i\right) \end{bmatrix} \\
\beta_{1_i} &= b_i \cdot u_{\max}\left(\varphi_x \cdot \cos(\theta) + \varphi_y \cdot \sin(\theta)\right) \\
\beta_{2_i} &= -\frac{w_{\max}}{\pi} \Delta_\theta\left(\theta_{nh_i}, \theta_i\right), \; i \in M_b \; {}^b\Delta_1 \leq 0 \\
\beta_{2_\lambda} &= -\frac{w_{\max}}{\pi} \Delta_\theta\left(\theta_{nh_\lambda}, \theta_\lambda\right), \; \lambda \in \left\{{}^b\mathrm{N}_{\rho_b}\right\} \; {}^b\Delta_1 > 0 \\
\beta_{2_j} &= w_{\max} b_j \cdot V_{\theta_j}, \; j \in \left\{M_b \backslash {}^b\mathrm{N}_{\rho_b}\right\} \; {}^b\Delta_1 > 0
\end{aligned} \tag{6}$$

is globally asymptotically stable.

PROOF. We form the control Lyapunov function: $V_a = V + \frac{1}{2}k_1 z_1^2 + \frac{1}{2}k_2 z_2^2$. Taking the time derivative: $\dot{V}_a = \dot{\mathbf{x}} \cdot \nabla V + k_1 z_1\left(-c_1 z_1 - \beta_1\right) + k_2 z_2\left(-c_2 z_2 - \beta_2\right)$. Substituting the proposed control laws, we get:

For ${}^b\Delta_1 \leq 0$, we have that $\dot{V}_a = {}^b\Delta_1 \leq 0$. A similar to Lemma 1 can now be stated:

Lemma 2. If ${}^b\Delta_1 > 0$ then $\exists i \in \{1, \ldots, 2^m\}$:${}^b\Delta_i < 0$.

PROOF. The proof is similar to the proof of Lemma 1 and is therefore omitted. □

For ${}^b\Delta_1 > 0$ then there is at least one j for which ${}^b\Delta_j < 0$ as we deduced from (Lemma 2) and thus $\rho_b \neq \{\emptyset\}$. We choose $j = \rho_b$ for the same reasons as in Proposition 1. Substituting the controls for this case, we get $\dot{V}_a = {}^a\Delta_{\rho_b} \leq 0$.

Now let $E = \{(\mathbf{x}, \mathbf{z}) : \dot{V}_a(\mathbf{x}, \mathbf{z}) = 0\}$ and $E \supset S = \{(\mathbf{x}, \mathbf{z}) : (\omega_i, u_i) = \mathbf{z}_i = \beta_i = (0, 0), \; \forall i \in M_b\}$ is an invariant set. From the proposed control law, it can be seen that $u_i = 0, \forall i \in M_b$ only at the destination, and for all other configurations the controller provides a direction of movement. According to LaSalle's invariance principle, the trajectories of the system converge asymptotically to the largest invariant set, which is the destination configuration $\square$

Proposition 4. The control law defined in Proposition 3, respects the input constraints defined in (2).

PROOF. Taking the absolute values, we get:

$$
\begin{aligned}
{}^{x}\xi_i' &= b_i \cdot |k_1 z_{1_i} + u_i| \le \\
&\le \frac{|k_1 z_{1_i}| + |u_i|}{1 + |k_1 z_{1_i}| + |k_2 z_{2_i}| + |u_i| + |w_i|} \le 1 \\
{}^{y}\xi_i' &= b_i \cdot |k_2 z_{2_i} + w_i| \le \\
&\le \frac{|k_2 z_{2_i}| + |w_i|}{1 + |k_1 z_{1_i}| + |k_2 z_{2_i}| + |u_i| + |w_i|} \le 1
\end{aligned}
$$

$\square$

6. SIMULATIONS

To verify the navigation properties of the methodology, we set up a simulation with four nonholonomic unicycles that are about to navigate from an initial to a final configuration, without hitting each other. The robots are placed at several initial configurations and the paths travelled are recorded and depicted in the figures that follow. The chosen configurations constitute non - trivial setups, since the straight paths connecting initial and final positions are obstructed by other robots.

In the first case (figure 1) the four robots were positioned at:

$$p^T = [0.4 \ \ -0.4 \ \ -0.4 \ \ -0.4 \ 0.4 \ 0.4 \ \ -0.4 \ 0.4]$$

with angles $[\theta_1 \ldots \theta_4] = [0\ 0\ 0\ 0]$ and their destination configuration was set at: $[{}^{d}q_1^T \ldots {}^{d}q_4^T] = [\ -0.4\ 0.4\ 0.4\ 0.4\ \ -0.4\ \ -0.4\ 0.4\ \ -0.4]$ with

$$[{}^{d}\theta_1 \ldots {}^{d}\theta_4] = [\,0\ 0\ 0\ 0\,]$$

. Figure (1a) denotes the initial (R1. . . R4) and target (T1. . . T4) configurations of the four robots. Figures (1b-1d) depict the trajectories of the robots. As can be seen, the multirobot navigation function successfully resolves all the proximity situations and the nonholonomic controller successfully steers the system to its destination.

In the second case (figure 2) the four robots were positioned at:

$$p^T = [0.2\ 0.0\ 0.0\ 0.10 \ \ -0.2\ 0.0\ 0.0 \ \ -0.1]$$

with angles $[\theta_1 \ldots \theta_4] = [0\ 0\ 0\ 0]$ and their destination configuration was set at: $[{}^{d}q_1^T \ldots {}^{d}q_4^T] = [\ -0.2\ 0.0\ 0.0\ 0.10\ 0.2\ 0.0\ 0.0\ \ -0.1]$ with

$$[{}^{d}\theta_1 \ldots {}^{d}\theta_4] = [\,0\ 0\ 0\ 0\,]$$

. Figure (2a) denotes the initial (R1. . . R4) and target (T1. . . T4) configurations of the four robots.

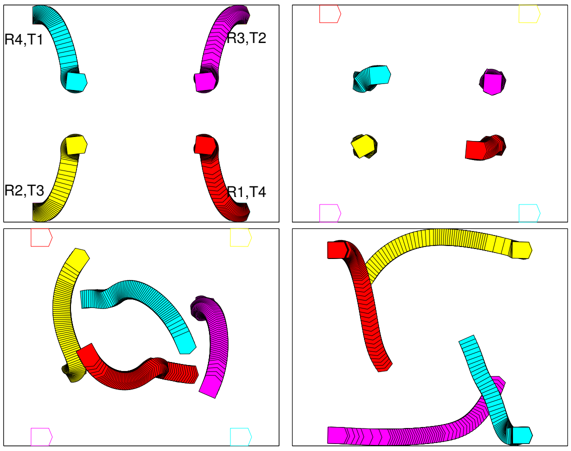

Fig. 1. (a) Initial Conf., (b,c) Intermediate Conf., (d) Intermediate and Final Configurations

Figures (2b-2d) depict the trajectories of the robots. As can be seen, the multirobot navigation function successfully resolves all the proximity situations and the nonholonomic controller successfully steers the system to its destination.

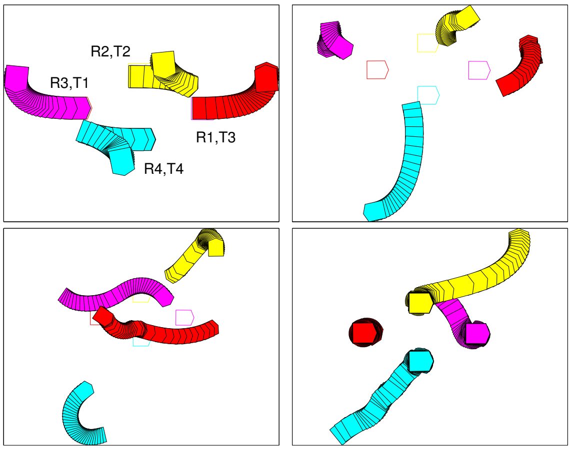

Fig. 2. (a) Initial Conf., (b,c) Intermediate Conf., (d) Intermediate and Final Configurations

7. CONCLUSIONS - ISSUES FOR FURTHER RESEARCH

In this paper we have successfully tackled the problem of input constraints in nonholonomic multirobot navigation scenarios. A new Dipolar Potential Field structure was introduced, which favors faster robot orientation close to the destination. A discontinuous backstepping controller was designed to suppress the chattering behavior. The derived Dipolar Multirobot Navigation Function (DMNF), along with the specially designed discontinuous feedback control law, provides guaranteed global convergence for the system, while respecting the input constraints. The methodology due its closed loop nature provides a robust navigation scheme with guaranteed collision avoidance and it's global convergence properties guarantee that a solution will be found if one exists. The closed form control law and the analytic expression of the potential function and its derivatives, provides fast feedback and makes the methodology particularly suitable for real time implementation while the input constraint controller renders the methodology easily realizable on actual hardware. The methodology can be easily applied to a three dimensional workspace and through proper transformations to arbitrarily shaped robots.

Current research directions are towards decentralized multiple robot navigation with limited workspace knowledge, limited vision capability, cooperation between mobile robots, formation control, as well as locomotion issues.

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