Closed Loop Navigation for Multiple Non - Holonomic Vehicles

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Abstract

In this paper we incorporate dipolar potential fields used for nonholonomic navigation into a novel potential function designed for multi – robot navigation. The derived navigation function is suitable for navigation of multiple nonholonomic vehicles. A properly designed discontinuous feedback control law is applied to steer the nonholonomic vehicles. The derived closed form control scheme provides robust navigation with guaranteed collision avoidance and global convergence properties, as well as fast feedback, rendering the methodology particularly suitable for real time implementation. Collision avoidance and global convergence properties are verified through non - trivial computer simulations.

1 Introduction

Multiple robot navigation is a research area with an increasing research interest over the last decade [24, 14, 11, 23, 16, 10]. In the last few years multi - robot navigation for Non - Holonomic vehicles is gaining increasing attention [19, 3, 6, 7].

Our main interest is to deduce global convergent control schemes with collision avoidance, suitable for real time implementation. Many researchers consider the local stabilization issues [25, 3, 5, 4] without any deadlock resolution mechanism. There are also several attempts to attack the problem with neural nets [26, 7] and with fuzzy logic controllers [6]. In [19] a global convergent algorithm is presented for nonholonomic path planning, based on probabilistic roadmaps, but the me thodology cannot be used for real time implementation due to its complexity.

Nonholonomic stabilization has attracted the attention of the control community over the years, due to the fact that nonholonomic systems do not satisfy the Brockett's necessary smooth feedback stabilization condition [2]. In this paper we address the problem of multiple nonholonomic robot navigation by constructing a potential function that can handle both multiple robot situations and provide feasible nonholonomic trajectories due to its dipolar structure.

The rest of the paper is organized as follows: Section 2 introduces the motivating problem. Section 3 outlines the concept of multiple robot navigation functions. Section 4 presents the discontinuous feedback control scheme. Section 5 presents simulation results for a number of non - trivial navigational tasks. Finally, section 6 summarizes the conclusions and indicates our current research directions.

2 Problem Statement

Consider the following system of m nonholonomic vehicles:

$$\begin{aligned} \dot{x}_i &= u_i \cdot \cos(\theta_i) \\ \dot{y}_i &= u_i \cdot \sin(\theta_i) \\ \dot{\theta}_i &= w_i \end{aligned} \tag{1}$$

with $i \in \{1 \ldots m\}$. (x_i, y_i, θ_i) are the position and orientation of each robot, u_i and w_i are the translational and rotational velocities respectively.

The problem can be now stated as follows: *"Given the nonholonomic system (1), derive a feedback kinematic control law that steers the system from any initial configuration to the goal configuration avoiding collisions. The environment is assumed perfectly known and stationary, while each robot acts as a potential obstacle to the others."*

3 Multi-Robot Navigation Functions

In a previous work [12] the authors presented an extension to the navigation function methodology with applications to multiple robot navigation. In this section we present how this novel class of potential functions can be enhanced with a dipolar structure [20] to provide trajectories suitable for nonholonomic navigation.

As it was shown in [12, 13] the function: $\varphi = \frac{\gamma_d}{\left(\gamma_d^k + G\right)^{1/k}}$ proposed by [9] for single robot navigation, with a proper selection of G can be used for multiple robot navigation and can be made a navigation function by an appropriate choice of k. Our assumption that we have spherical robots and spherical obstacles does not constrain the generality of this work since it has been proven [9] that navigation properties are invariant under diffeomorphisms. Methods for constructing analytic diffeomorphisms are discussed in [18, 17] for point robots and in [21, 22]for rigid body robots.

Let us assume the following situation: We have m mobile robots, and their workspace $W \subset R^2$. Each robot R_i, $i = 1 \ldots m$ occupies a disk in the workspace: $R_i = \{q \in R^2 : \|q - q_i\| \leq r_i\}$ where $q_i \in R^2$ is the center of the disk and r_i is the radius of the robot. The position vector of the robots is represented by $q = [q_1 \ldots q_m]$. The orientation vector of the robots is represented by $\theta = [\theta_1 \ldots \theta_m]$ where θ_i represents the orientation of each robot . The configuration of each robot is then represented by $p_i = \left[\begin{array}{cc} q_i & \theta_i \end{array}\right] \in R^2 \times (-\pi, \pi]$ and the configuration space C is spanned by $p = \left[q_1^T \ldots q_m^T \; \theta_1 \ldots \theta_m\right]^T$.

3.1 Mathematical Tools - Terminology

The robot proximity functions, a measure for the distance between two robots i and j, are defined by: $\beta_{i,j}(q) = q^T D_{ij} q - (r_i + r_j)^2$, where r_i is the radius of the i'th robot and D_{ij} is defined in [12]. We will use the term '**relation**' to describe the possible collision schemes that can be defined in a multi robot - obstacles scene. The '**set of relations**' between the members of a set can be defined as the set of all possible collision schemes between the members. A **binary relation** is a relation between two robots. Any relation can be expressed as a set of binary relations. A '**relation tree**' is the set of robots-obstacles that form a linked team. Each *relation* may consist of more than one tree (figure 1). We will call the number of binary relations in a relation, the '**relation level**'.

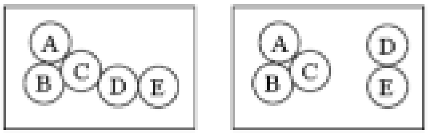

Figure 1 *: (a) One – tree relation, (b) Two tree relation*

A **relation proximity function (RPF)** provides a measure of the distance between the robots involved in a relation. Each relation has it's own *RPF*. An *RPF* assumes the value of zero whenever the related robots collide and increases wrt the distance of the related robots:

$$b_R = q^T \cdot P_R \cdot q - \sum_{\{i,j\}\in R} (r_i + r_j)^2$$

where R is the set of binary relations (e.g. for the relation in figure (1.b) $R = \{\{A,B\},\{A,C\},\{B,C\},\{D,E\}\}$) and $P_R = \sum\limits_{\{i,j\}\in R} D_{i,j}$ is the **relation matrix** of *RPF*. The gradient and Hessian of the *RPF* are: $\nabla b_R = 2P_R \cdot q$ and $\nabla^2 b_R = 2P_R$.

A **Relation Verification Function (RVF)** is defined by:

$$g_{R_j}\left(b_{R_j}, B_{R_j^C}\right) = b_{R_j} + \lambda \cdot b_{R_j} \Big/ \left(b_{R_j} + B_{R_j^C}^{1/h}\right) \tag{2}$$

where $\lambda, h > 0$, R_j^C is the complementary to R_j set of *relations* in the same level, j is an index number defining the relation in the level and $B_{R_j^C} = \prod\limits_{k\in R_j^C} b_k$. An *RVF* is zero if a relation holds while no other relation from the same level holds and has the properties: (a) $\lim\limits_{x\to 0}\lim\limits_{y\to 0} g_x(x,y) = \lambda$, (b) $\lim\limits_{y\to 0}\lim\limits_{x\to 0} g_x(x,y) = 0$.

Based on the above properties, in a robot proximity situation, one can verify that: if $\left(g_{R_j}\right)_k = 0$ at some *level* k then $\left(g_{R_i}\right)_h \neq 0$ for any *level* h and $i \neq j$ in level k . It should be noted hereby that since in the highest relation level only one relation exists, there will be no complementary relations and the RVF will be identical to the RPF e.g. $\lambda = 0$ for this relation. We can now define

$$G = \prod_{L=1}^{n_L} \prod_{j=1}^{n_{R,L}} \left(g_{R_j}\right)_L$$

with n_L the number of *levels* and n_R, L the number of *relations* in *level* L. Figure (2) demonstrates several types of relations of a four – member team.

The gradient and the Hessian of $g_x(x,y)$ are given below:

$$\nabla g\left(b,\tilde{b}\right) = \nabla b \cdot \left(1 + \frac{\lambda}{b + \tilde{b}^{1/h}}\right) - \frac{b\cdot\lambda}{\left(b+\tilde{b}^{1/h}\right)^2}\left(\nabla b + \nabla \tilde{b}^{1/h}\right) \tag{3}$$

$$\begin{aligned}\nabla^2 g\left(b,\tilde{b}\right) = {} & \left(1 + \tfrac{\lambda}{b+\tilde{b}^{1/h}}\right)\cdot \nabla^2 b - \tfrac{2\lambda}{\left(b+\tilde{b}^{1/h}\right)^2} \nabla b \cdot \left(\nabla b + \nabla \tilde{b}^{1/h}\right)^T + \\ & b\cdot\lambda\left(\tfrac{2}{\left(b+\tilde{b}^{1/h}\right)^3}\left(\nabla b + \nabla\tilde{b}^{1/h}\right)\left(\nabla b + \nabla\tilde{b}^{1/h}\right)^T - \tfrac{1}{\left(b+\tilde{b}^{1/h}\right)^2}\left(\nabla^2 b + \nabla^2\tilde{b}^{1/h}\right)\right)\end{aligned} \tag{4}$$

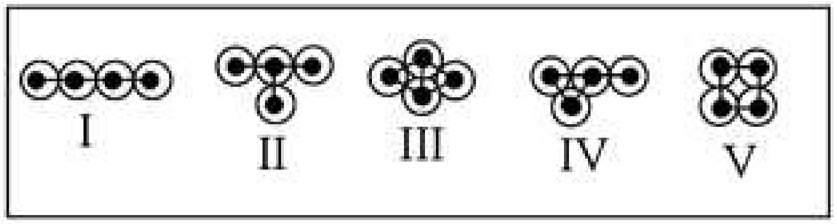

Figure 2 *: I, II are level 3; IV, V are level 4 and III is a level 5 relation*

3.2 Dipolar Navigation Functions

To be able to produce a dipolar potential field, φ must be modified as follows:

$$\varphi = \frac{\gamma_d}{\left(\gamma_d^k + H_{nh} \cdot G\right)^{1/k}} \tag{5}$$

where H_{nh} has the form of a pseudo - obstacle. A possible selection of H_{nh} would be:

$$H_{nh} = \varepsilon_{nh} + \left(\prod_{i=1}^{m} \eta_{nh_i}\right)^{\mu}$$

with $\eta_{nh_i} = \left\|(q - q_d) \cdot \mathbf{n}_{d_i}\right\|^2$, where $\mathbf{n}_{d_i} = \left[O_{1x2(i-1)} \ \cos\left(\theta_{d_i}\right) \ \sin\left(\theta_{d_i}\right) \ O_{1x2(m-i)}\right]^T$ and μ a tuning parameter. Subscript d denotes destination. Moreover $\gamma_d = \|p - p_d\|^2$, i.e. the angle is incorporated in the distance to the destination metric. As is shown in the ongoing analysis, the proposed modifications of the potential function does not affect its navigation properties, as long as the workspace is bounded and $\varepsilon_{nh} > \varepsilon\left(k\right)$.

3.3 Proof of correctness

Let $\varepsilon > 0$. Define $B_i^l\left(\varepsilon\right) = \left\{q : 0 < \left(g_{R_i}\left(q\right)\right)_l < \varepsilon\right\}$. We can then discriminate the following topologies:

1. The destination point q_d
2. The free space boundary: $\partial F\left(q\right) = G^{-1}\left(\delta\right)$, $\delta \to 0$
3. The robot/obstacle proximity set:$F_0\left(\varepsilon\right) = \bigcup\limits_{L=1}^{n_L} \bigcup\limits_{i=1}^{n_{R,L}} B_i^L\left(\varepsilon\right) - \left\{q_d\right\}$, with n_L and n_R, L as defined above.
4. The robot/obstacle distant set: $F_1\left(\varepsilon\right) = F - \left(\left\{q_d\right\} \bigcup F_0\left(\varepsilon\right)\right)$

Proposition 1. *The destination point q_d is a non – degenerate local minimum of φ.*

Proof. Similar to this found in [9]. From eq. (5), we have:

$$\begin{aligned}\nabla\varphi\left(q_d\right) = & \frac{1}{\left(\gamma_d^k + H_{nh}\cdot G\right)^{2/k}} \\ & \left(\left(\gamma_d^k + H_{nh}\cdot G\right)^{1/k}\nabla\gamma_d - \gamma_d\nabla\left(\gamma_d^k + H_{nh}\cdot G\right)^{1/k}\right) \\ & = 0\end{aligned}$$

since at q_d both γ_d and $\nabla\gamma_d$ are zero. The Hessian at a critical point is:

$$\begin{aligned}\nabla^2\varphi = & \frac{1}{\left(\gamma_d^k + H_{nh}\cdot G\right)^{2/k}}\cdot \\ & \cdot\left(\left(\gamma_d^k + H_{nh}\cdot G\right)^{1/k}\nabla^2\gamma_d - \gamma_d\nabla^2\left(\gamma_d^k + H_{nh}\cdot G\right)^{1/k}\right)\end{aligned}$$

but at qd, $\nabla^2\gamma_d = 2I$ and the Hessian reduces to:

$$\left(\nabla^2\varphi\right)\left(q_d\right) = 2\left(H_{nh}\cdot G\right)^{-1/k} I$$

which is non – degenerate since $H_{nh} > 0$. ☐

Proposition 2. *All the critical points are in the interior of the free space.*

Proof. Let q_0 be a point on ∂F and suppose that $\left(g_{R_j}\right)_\kappa\left(q_0\right) = 0$ for the relation j of level k. Then $\left(g_{R_i}\right)_h\left(q_0\right) > 0$, for any *level* h and $i \neq j$ in level k, because only one *RVF* can hold at a time. Then at q_0:

$$\begin{aligned}\nabla\varphi\left(q_0\right) = & \frac{1}{\left(\gamma_d^k + H_{nh}\cdot G\right)^{\frac{2}{k}}}\cdot \\ & \cdot\left(\left(\gamma_d^k + H_{nh}\cdot G\right)^{\frac{1}{k}}\nabla\gamma_d - \gamma_d\nabla\left(\gamma_d^k + H_{nh}\cdot G\right)^{\frac{1}{k}}\right)\Big|_{q_0} \\ & = -H_{nh}\cdot\frac{1}{k}\gamma_d^{-k}\left(\prod_{L=1}^{n_L}\prod_{\substack{i(L)=1 \\ i(k)\neq j}}^{n_{R,L}}\left(g_{R_i}\right)_L\right)\cdot\nabla\left(g_{R_j}\right)_k \\ & \neq 0\end{aligned}$$

since $H_{nh} \neq 0$

☐

Proposition 3. *For every* $\varepsilon > 0$ *, there exists a positive integer* $\mathrm{N}(\varepsilon)$ *such that if* $k > \mathrm{N}(\varepsilon)$ *then there are no critical points of* $\hat{\varphi}$ *in* $F_1(\varepsilon)$ *.*

Proof. Similar to this found in [9]. As is discussed in [9], the functions

$$\hat{\varphi} = \frac{\gamma_d^k}{S} \tag{6}$$

and eq. (5), where $S = H_{nh} \cdot G$, have identical navigation properties. Taking the gradient of $\hat{\varphi}$, we have:

$$\nabla\hat{\varphi} = \frac{1}{S^2}\left(Sk\gamma_d^{k-1}\nabla\gamma_d - \gamma_d^k\nabla S\right)$$

At a critical point it will be:

$$\gamma_d\nabla S = Sk\nabla\gamma_d$$

Taking the magnitude of both sides:

$$2\kappa S = \sqrt{\gamma_d}\,\|\nabla S\|$$

because $\|\nabla\gamma_d\| = 2\sqrt{\gamma_d}$. A sufficient condition for the above equality not to hold is:

$$\kappa > \frac{1}{2}\frac{\sqrt{\gamma_d}\,\|\nabla S\|}{S}$$

for all

$$q \in F_1(\varepsilon)$$

An upper bound for the right side of the inequality can be derived, provided that the workspace (or configuration space) C is bounded and is given by:

$$\begin{aligned}&\tfrac{1}{2}\tfrac{\sqrt{\gamma_d}\|\nabla S\|}{S} < \\ &< \tfrac{1}{2}\tfrac{1}{\varepsilon}\max_C\left\{\sqrt{\gamma_d}\right\}\cdot\left(\max_C\{\|\nabla H_{nh}\|\} + \sum_{L=1}^{n_L}\sum_{j=1}^{n_{R,L}}\max_C\left\{\left\|\nabla\left(g_{R_j}\right)_L\right\|\right\}\right) \triangleq N(\varepsilon)\end{aligned}$$

since $\left(g_{R_j}\right)_L > \varepsilon, j \in \{0..n_{R,L}\}, L \in \{0..n_L\}$. For the above inequality to be true, an appropriate choice of ε_{nh} must be made. So for any $\varepsilon_{nh} \geq \varepsilon$ the above inequality holds. $\square$

Hence the set away from the obstacles is 'cleaned' from critical points. The workspace can be bounded with several obstacles prohibiting the motion of robots beyond them or by defining a world obstacle in the sense of robot proximity function: $\beta_{w,i} = (-1)\left(q_i^T q_i - (r_w - r_i)^2\right)$ where the index i refers to the robot and the index w refers to the world obstacle.

Proposition 4. *There exists an* $\varepsilon_0 > 0$ *, such that* $\hat{\varphi}$ *has no local minimum in* $F_0(\varepsilon)$*, as long as* $\varepsilon < \varepsilon_0$*.*

Proof. Since we can express $S = H_{nh} \cdot G$ as $S = g_i \cdot \bar{g}_i$, due to $H_{nh} > \varepsilon$, the proof is the same as in [13] if we substitute G with S and express $S = g_i \cdot \bar{g}_i$. ☐

Proposition 5. *There exists* $\varepsilon_1 > 0$ *and* $h_1 > 0$*, such that the critical points of* $\hat{\varphi}$ *are non-degenerate as long as* $\varepsilon < \varepsilon_1$ *and* $h > h_1$ *(Morse Property)*

Proof. Since we can express $S = H_{nh} \cdot G$ as $S = g_i \cdot \bar{g}_i$, due to $H_{nh} > \varepsilon$, the proof is the same as in [13] if we substitute G with S and express $S = g_i \cdot \bar{g}_i$. ☐

4 Non - Holonomic Control

In the following analysis we will use V for denoting the navigation function instead of φ for notational consistency.

Define $M = \{1, \ldots, m\}$ and $\Omega = P(M)$ where P denotes the power set operator. Assuming that Ω is an ordered set, let N_j denote the j 'th element of Ω where $j \in \{1, \ldots, 2^m\}$. Then $N_j \subseteq M$ with $N_1 = \{\emptyset\}$ and $N_{2^m} = M$. We can now define:

$$\Delta_j = K_\theta \cdot \sum_{i \in \{M \backslash N_j\}} \left(V_{\theta_i} \cdot (\theta_{nh_i} - \theta_i)\right) - K_u \sum_{i=1}^{m} \left(\left|V_{x_i} \cdot \cos(\theta_i) + V_{y_i} \cdot \sin(\theta_i)\right| \cdot Z_i\right) - K_\theta \cdot \sum_{i \in N_j} V_{\theta_i}^2 \quad (7)$$

with

$$Z_i = K_u \cdot \left(V_{x_i}^2 + V_{y_i}^2\right) + K_z \left((x_i - x_{d_i})^2 + (y_i - y_{d_i})^2\right)$$

and

$$\theta_{nh_i} = atan2\left(V_{y_i} \cdot side_i, V_{x_i} \cdot side_i\right)$$

with $side_i = sgn\left((q - q_d) \cdot \mathbf{n}_{d_i}\right)$. V_q denotes the derivative $\frac{\partial V}{\partial q}$ of V along q. K_θ, K_u are positive constants. Define $H = \{j : \Delta_j < 0\}$ and $\rho = \left\{j : \Delta_j = \max_{i \in H}(\Delta_i)\right\}$. We can now state the following:

Proposition 6. *The system (1) under the control law:*

$$\begin{array}{l} \omega_i = K_\theta \cdot (\theta_{nh_i} - \theta_i), \ i \in M \ \ \Delta_1 \leq 0 \\ \omega_l = K_\theta \cdot (\theta_{nh_l} - \theta_l), \ l \in \{N_\rho\}, \ \Delta_1 > 0 \\ \omega_j = -K_\theta \cdot V_{\theta_j}, \ j \in \{M \backslash N_\rho\}, \ \Delta_1 > 0 \end{array}$$

$$u_i = -\mathrm{sgn}\left(V_{x_i} \cdot \cos\left(\theta_i\right) + V_{y_i} \cdot \sin\left(\theta_i\right)\right) \cdot Z_i, \qquad i \in M$$

is globally asymptotically stable.

Proof. The navigation function V studied in the previous section serves as a Lyapunov function candidate. We will now examine the derivative of V along the trajectories of (1):

$$\dot{V} = \frac{\partial V}{\partial t} + \nabla V \cdot \dot{\mathbf{x}} = \nabla V \cdot \dot{\mathbf{x}}$$

since $V = V\left(x\right)$ with $\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \; \dot{y}_1 \; \dot{\theta}_1 \; \ldots \; \dot{x}_m \; \dot{y}_m \; \dot{\theta}_m \end{bmatrix}^T$ and

$$\nabla V = \begin{bmatrix} \frac{\partial V}{\partial x_1} \; \frac{\partial V}{\partial y_1} \; \frac{\partial V}{\partial \theta_1} \; \ldots \; \frac{\partial V}{\partial x_m} \; \frac{\partial V}{\partial y_m} \; \frac{\partial V}{\partial \theta_m} \end{bmatrix}^T$$

Substituting we get:

$$\dot{V} = \sum_{i=1}^{m} \left(\frac{\partial V}{\partial x_i}\dot{x}_i + \frac{\partial V}{\partial y_i}\dot{y}_i + \frac{\partial V}{\partial \theta_i}\dot{\theta}_i \right) = \sum_{i=1}^{m} \left(u_i \left(V_{x_i} \cdot \cos\left(\theta_i\right) + V_{y_i} \cdot \sin\left(\theta_i\right)\right) + \dot{\theta}_i V_{\theta_i} \right)$$

We are interested in establishing that $\dot{V} < 0$ almost everywhere, and the sets of points where $\dot{V} = 0$ except from the destination are not invariant. Applying the proposed controls, we get:

For $\Delta_1 \leq 0$ we have:

$$\omega_i = K_\theta \cdot \left(\theta_{nh_i} - \theta_i\right), \; i \in M$$

$$u_i = -\mathrm{sgn}\left(V_{x_i} \cdot \cos\left(\theta_i\right) + V_{y_i} \cdot \sin\left(\theta_i\right)\right) \cdot Z, \; i \in M$$

Then $\dot{V} = \Delta_1 \leq 0$. To proceed with the proof we will need the following lemma:

Lemma 1. *If* $\Delta_1 > 0$ *then* $\exists i \in \{1, \ldots, 2^m\} : \Delta_i < 0$

Proof. If $\Delta_1 > 0$ then since:

$$-K_u \sum_{i=1}^{m} \left(\left|V_{x_i} \cdot \cos\left(\theta_i\right) + V_{y_i} \cdot \sin\left(\theta_i\right)\right| \cdot Z_i\right) \leq 0$$

It must be

$$K_\theta \cdot \sum_{i=1}^{m} \left(V_{\theta_i} \cdot \left(\theta_{nh_i} - \theta_i \right) \right) > 0$$

which means that there exists at least one k for which $V_{\theta_k} \neq 0$ and the term $-K_\theta \cdot \sum\limits_{i \in N_j} V_{\theta_i}^2$ of some Δ_i will be negative definite. For the worst case scenario, $\Delta_{2^m} < 0$ since $N_{2^m} = M$. $\square$

For $\Delta_1 > 0$ then there is at least one j for which $\Delta_j < 0$ as we deduced from (Lemma 1) and thus $\rho \neq \{\emptyset\}$. We choose $j = \rho$ because we want the maximum possible number of robots to follow the dipole generated Non-Holonomic trajectories. The rest will be doing a conflict avoidance manoeuver. The controls in those cases take the form:

$$\begin{array}{l} \omega_l = K_\theta \cdot \left(\theta_{nh_l} - \theta_l \right), \ l \in \{N_\rho\}, \ \Delta_1 > 0 \\ \omega_j = -K_\theta \cdot V_{\theta_j}, \ j \in \{M \backslash N_\rho\}, \ \Delta_1 > 0 \end{array}$$

$$u_i = -\mathrm{sgn}\left(V_{x_i} \cdot \cos\left(\theta_i\right) + V_{y_i} \cdot \sin\left(\theta_i\right) \right) \cdot Z_i, i \in M$$

Then $\dot{V} = \Delta_\rho \leq 0$

Now let $E = \{\mathbf{x} : \dot{V}(\mathbf{x}) = 0\}$ and $E \supset S = \{x : \omega_i = u_i = 0, \forall i \in M\}$ is an invariant set. From the proposed control law, it can be seen that $u_i = 0, \forall i \in M$ only at the destination, and for all other configurations the controller provides a direction of movement. According to LaSalle's invariance principle, the trajectories of the system converge asymptotically to the largest invariant set, which is the destination configuration $\square$

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