Prestress Losses

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Losses Due to Elastic Deformation of Pre-Tensioned Concrete

Consider a pre-tensioned beam with a tendon at the c.g.c:

\[ \varepsilon_{ES} = \frac{\Delta_{ES}}{L} \]
For the pre-tensioned tendon:

\[ \varepsilon_{KS} = \frac{\sigma_c(t)}{E_c} = \frac{P_i}{A_c \cdot E_{cm}(t)} \]

\[ \Delta P_{el} = A_p \cdot \frac{E_p}{E_{cm}(t)} \cdot \frac{P_i}{A_c} \]

\[ \Delta P_{el} = A_p \cdot \frac{E_p}{E_{cm}(t)} \cdot \Delta \sigma_c(t) \]

For an Eccentric Tendon and Accounting Self-Weight

From stress superposition the stress at the c.g.s is:

\[ \Delta \sigma_c(t) = -\frac{P}{A_c} - \frac{P \cdot y_p \cdot y_{cs}}{I_c} + \frac{M_{cm} \cdot y_{cs}}{I_c} \]

\( P_i \) has a lower value after transfer of prestress compared to the initial jacking force \( P_j \) (due to elastic shortening)

\( P_i \) is difficult to accurately determine and the reduction is only a small percentage of \( P_i \)

Therefore assume \( P_i = P_j \)
Example

![Diagram of a beam with dimensions and prestressing tendons]

Given:
- $f_{ck} = 41.4$ MPa, $f_{ci} = 31$ MPa
- $f_{pk} = 1862$ MPa, $f_{p0,1k} = 1551.7$ MPa, $E_p = 186.2$ GPa
- $b = 381$ mm, $h = 762$ mm
- Ten 12.7 mm tendons are used to prestress the beam ($A_{	ext{tendon}} = 98.7$ mm$^2$)

Find:
(a) Prestress loss due to elastic shortening
(b) Check stresses in concrete

\[ A_p = 10 \cdot 98.7\text{mm}^2 = 987\text{mm}^2 = 0.000987\text{m}^2 \]

\[ \sigma_{p,\text{min}} = \min\{0.8 \cdot f_{ck}, 0.9 \cdot f_{p0,1k}\} = \min\{0.8 \cdot 1862, 0.9 \cdot 1551.7\} = 1396.5\text{MPa} \]

\[ P_i = A_p \cdot \sigma_{p,\text{max}} = 0.000987 \text{m}^2 \cdot 1396.5\text{MPa} = 1378\text{kN} \]

\[ A_i = 0.762\text{m}^2 \cdot 0.381\text{m}^2 = 0.2903\text{m}^2 \]

\[ I_y = \frac{0.381\text{m} \cdot (0.762\text{m})^3}{12} = 0.01405\text{m}^4 \]

\[ y_p = \frac{0.762\text{m}}{2} - 0.102\text{m} = 0.279\text{m} \]
Solution

\[ w_{sw} = 0.762m \cdot 0.381m \cdot \left(23.5 \frac{kN}{m^3}\right) = 6.83 \frac{kN}{m} \]

\[ M_{sw} = \frac{w_{sw} \cdot L^2}{8} = \frac{6.83 \cdot (15.2)^2}{8} = 197.2kNm \]

\[ y_p = y_{cs} \]

**Stress at centroid of tendon at mid-span:**

\[
\Delta \sigma_c(t) = \frac{1378}{0.2903} - \frac{1378 \cdot 0.279}{0.01405} - \frac{197.2 \cdot 0.279}{0.01405} = -8465kPa = -8.47MPa
\]

\[ E_c = 22 \left(\frac{f_c}{10}\right)^\frac{3}{7} = 22 \left[\frac{31}{10}\right]^\frac{3}{7} = 30.89GPa \]

\[ \Delta P_{cl} = A_p \cdot \frac{E_p}{E_{cs}(t)} \cdot \Delta \sigma_c(t) = 0.000987 \cdot \frac{186.2}{30.89} \cdot 8470 = 50.4kN \]

**Force in tendon after elastic shortening of concrete:**

\[ P_{t,0} = P_t - \Delta P_{cl} = 1378 - 50.4 = 1327.6kN \]

**Stresses at top and bottom immediately after prestress:**

\[
\sigma_{top} = -\frac{P_{t,0}}{A_c} \cdot \frac{y_p \cdot y_b}{I_c} + \frac{M_{sw} \cdot y_b}{I_c}
\]

\[ \sigma_{top} = -\frac{1327.6}{0.2903} \cdot \frac{0.279 \cdot 0.381}{0.01405} + \frac{197.2 \cdot 0.381}{0.01405} = -9270kPa = -9.27MPa
\]
Allowable stresses as per EC2:
\[ \sigma_{ci} \leq 0.6 \cdot f_{c} \]
\[ \sigma_{c} \leq 0.6 \cdot 31 \]
\[ \sigma_{c} \leq 18.6 \text{MPa} \]
\[ \sigma_{bot} = -9.27 \text{MPa} \Rightarrow OK \]
\[ \sigma_{top} = 0.124 \text{MPa} \Rightarrow OK \]

What if the Beam was Post-Tensioned?

Same data as before but for post-tensioned beams EC2:

\[
\Delta P_{el} = A_p \cdot E_p \sum \left[ j \cdot \frac{\Delta \sigma(t)}{E_{cm}(t)} \right] \]

\[
\Delta P_{el} = 0.000987 \cdot 186200 \left( \frac{0.5 \cdot 8.47}{30.89} \right) = 25.2 \text{kN} \]

\[
j = \frac{n-1}{2n} = \frac{10-1}{2 \cdot 10} = 0.45 \]

\[
\Delta P_{el} = 0.000987 \cdot 186200 \left( \frac{0.45 \cdot 8.47}{30.89} \right) = 22.7 \text{kN} \]

The use of 0.5 instead of the actual value for j is conservative.
Loss Due to Anchorage Slip

Data on anchorage slip must be supplied by the manufacturer. For wedge type strand anchors the slip is 3 to 10 mm.

\[ \Delta P_{sl} = A_p \cdot E_p \cdot \frac{\Delta A}{L} \]

\( \Delta A \) is the anchor slip

L is the length of the tendon

Losses due to Friction

\[ \Delta P_{\mu} (x) = P_{\text{max}} \left( 1 - e^{-\mu(\theta + kx)} \right) \]

In the absence of data in a European Technical Approval, values for unintended regular displacements for internal tendons will generally be in the range \(0.005 < k < 0.01\) per metre.

Table 5.1: Coefficients of friction \(\mu\) of post-tensioned internal tendons and external unbonded tendons

<table>
<thead>
<tr>
<th></th>
<th>Internal tendons</th>
<th>External unbonded tendons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steel duct/ non</td>
<td>HDPE duct/ non</td>
</tr>
<tr>
<td></td>
<td>lubricated</td>
<td>lubricated</td>
</tr>
<tr>
<td>Cold drawn wire</td>
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<td>0.25</td>
</tr>
<tr>
<td>0.19</td>
<td>0.24</td>
<td></td>
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<tr>
<td>0.14</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>0.16</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strand</td>
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<td>0.24</td>
</tr>
<tr>
<td>0.65</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Deformed bar</td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>Smooth round bar</td>
<td>0.33</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: HPDE - High density polyethylene

for tendons which fill about half of the duct
**Example for Friction Losses**

Post-tensioned concrete beam with the tendon profile shown

Assume:
- \( P_{\text{max}} = 1400 \text{ MPa} \), \( k = 0.005 \)
- Internal tendons (7-wire strands)

Find:
- Prestress losses due to friction.

\[
\Delta P_{\mu}(x) = P_{\text{max}} \left(1 - e^{-\mu(\theta+kx)}\right)
\]

\[
\Delta P_{\mu}(x) = 1400(1 - e^{-0.19(0.267+0.005(21.41))}) = 96.04kN
\]

The friction loss from A to E is approximately -7%