Solidification problems

1. In the casting of steel under certain mold conditions, the mold constant in Chvorinov’s Rule is known to be 4.0 min/cm², based on previous experience. The casting is a flat plate (fig. 1) whose length \( l = 30 \text{ cm} \), width \( w = 10 \text{ cm} \), and thickness \( h = 20 \text{ mm} \). Determine how long it will take for the casting to solidify.

![Flat plate](image)

Area \( A = 2(lh) + 2(lw) + 2(wh) \)

Volume \( V = l \times h \times w \)

**Figure 1**

**Solution:**

Area \( A = 2(30 \times 10) + 2(30 \times 2) + 2(10 \times 2) = 760 \text{ cm}^2 \)

Volume \( V = 30 \times 10 \times 2 = 600 \text{ cm}^3 \)

Chvorinov’s Rule: \( T_{TS} = C_m \left( \frac{V}{A} \right)^2 = 4(600/760)^2 = 2.49 \text{ min} \)
2. A **cylindrical-shaped part** (fig. 2) is to be cast out of aluminum. The radius of the cylinder \( r = 250 \text{ mm} \) and its thickness \( h = 20 \text{ mm} \). If the mold constant \( C_m = 2.0 \text{ sec/mm}^2 \) in Chvorinov's Rule, how long will it take the casting to solidify?

![Cylindrical Volume and Area](image)

**Figure 2**

**Solution:**

Area \( A = \pi r^2 + 2 \pi rh = 2 \pi (250)^2 + 2 \pi (250) (20) = 424,115 \text{ mm}^2 \)

Volume \( V = \pi r^2 h = \pi (250)^2 (20) = 3,926,991 \text{ mm}^3 \)

Chvorinov's Rule: \( T_{TS} = C_m \frac{(V/A)^2}{2} = 2 \frac{(3,926,991 / 424,115)^2}{2} = 171.5 \text{ s} = 2.86 \text{ min} \)
3. In casting experiments performed using a certain alloy and type of sand mold, it took 155 sec for a cube-shaped casting to solidify. The cube was 50 mm on a side.

(a) Determine the value \( C_m \) of the mold constant in Chvorinov’s Rule.

(b) If the same alloy and mold type were used, find the total solidification time \( T_{TS} \) for a cylindrical casting in which the diameter \( r = 15 \text{ mm} \) and length \( h = 50 \text{ mm} \).

\[
\text{Cube \ (L=W=H)} \\
\begin{array}{c}
\text{Area} \\
A = 2lh + 2lw + 2wh \\
\text{Volume} \\
V = l \times h \times w
\end{array}
\]

\[
\text{Surface Area} = 2\pi r^2 + 2\pi rh \\
\text{Volume} = \pi r^2 h
\]

**Solution:**

(a) Area \( A = 6 \times (50)^2 = 15,000 \text{ mm}^2 \)

Volume \( V = (50)^3 = 125,000 \text{ mm}^3 \)

\( \frac{V}{A} = \frac{125,000}{15,000} = 8.333 \text{ mm} \)

\( C_m = \frac{T_{TS}}{(V/A)^2} = \frac{155}{(8.333)^2} = 2.232 \text{ s/mm}^2 \)
(b) Cylindrical casting with \( r = 15 \text{ mm} \) and \( h = 50 \text{ mm} \).

Area \( A = 2\pi r^2 + 2\pi rh = 2\pi (15)^2 + 2\pi (15)(50) = 6126 \text{ mm}^2 \)

Volume \( V = \pi r^2 h = \pi (15)^2 (50) = 35,343 \text{ mm}^3 \)

\[ \frac{V}{A} = \frac{35,343}{6126} = 5.77 \]

\( T_{TS} = 2.232 (5.77)^2 = 74.3 \text{ s} = 1.24 \text{ min.} \)

**Sphere**

- Surface area = \( 4\pi r^2 \)
- Volume = \( \frac{4}{3}\pi r^3 \)