**Synchronous Machines**

1.1 Introduction:
In this report the characteristics of synchronous machines, as well as their contribution to power systems operation are discussed.
A synchronous machine is an a.c. machine in which the rotor moves at a speed, which bears a constant relationship to the frequency of the current in the armature winding. As a motor, the shaft speed must remain constant irrespective of the load, provided that the supply frequency remains constant. As a generator, the speed must remain constant if the frequency of the output is not to vary. The field of a synchronous machine is a steady one. In very small machines this field may be produced by permanent magnets, but in most cases the field is excited by a direct current obtained from an auxiliary generator, which is mechanically coupled to the shaft of the main machine.

2 Synchronous machine characteristics and analysis:

2.1 Types of Synchronous machines
The armature or main winding of a synchronous machine may be on either the stator or the rotor. The difficulties of passing relatively large current at high voltages across moving contacts have made the stator wound armature the common choice for large machines. Stator-wound armature machines fall into two classes: (a) salient-pole rotor machines, and (b) non-salient-pole, or cylindrical-rotor, machines. The salient-pole machine has concentrated field windings and generally is cheaper than the cylindrical-rotor machine when the speed is low, (less than 1,500 rev/min). Salient-pole alternators are generally used when the prime mover is a water turbine or a reciprocating engine. In the round or cylindrical rotor case, the field winding is placed in slots along the rotor length. The diameter is relatively small (1-1.5 m) and the machine is suitable for operation at high speeds (3000 or 3600 rpm) driven by a steam or gas turbine. Hence it is known as a turbo generator.
The frequency of the generated e.m.f, and speed are related by:
\[ F = \frac{np}{60} \]
Where \( n \) is speed in rpm, and \( p \) is the number of pairs of poles.
A hydraulic turbine rotating at 50-300 rpm, depending on type. Thus needs many pole pairs to generate at normal frequencies.

2.2 Equivalent Circuit:
Whether a synchronous machine has a cylindrical or salient-pole rotor, its action is the same when generating power and can be best understood in terms of the representation shown in Figure 2.2. The equivalent circuit of a synchronous machine contains a voltage source \( E_F \) which is constant for a constant excitation current \( I_F \) and a series-connected reactance \( X_a \). In addition, an actual machine-winding will have resistance \( R \) and leakage reactance \( X_l \). The machine terminal voltage \( V \) is obtained from \( E_r \) by recognizing that the stator phase windings have a small resistance \( R_L \) and a leakage reactance \( X_l \) (about 10 per cent of \( X_a \)) resulting from flux produced by the stator but not crossing the air gap. The reactances \( X_L \) and \( X_a \) are usually considered together as the synchronous reactance \( X_s \),
The machine phasor equation is:

\[ EF = V + I \times Z_s \]  

(2.20)

where \( Z_s \) is the synchronous impedance

\[ Z_s = R + j(X_l + X_a) \]  

(2.21)

or

\[ Z_s = R + jX_s \]  

(2.22)

Where \( X_s \) is the synchronous reactance:

\[ X_s = X_a + X_l \]  

(2.23)

In Polar form the synchronous impedance is:

\[ Z_s = Z_s \angle \psi \]  

(2.24)

where:

\[ \psi = \tan^{-1} \frac{X_s}{R} \]  

(2.25)

and

\[ Z_s = \sqrt{R^2 + X_s^2} \]  

(2.26)

Frequently in synchronous machines \( X_s \ll R \), in which case equation (2.26) becomes

\[ Z_s = X_s \angle 90 = jX_s \]  

(2.27)

In the case of a synchronous motor, the current is entering the positive terminal. According to Kirchhoff’s law, equation (2.20) will not be valid for a motor, where:

\[ E = V - I \times Z_s \]  

(2.27)

### 2.3 M.M.F. wave diagrams of a synchronous machine:

The operation of a synchronous machine may be understood by considering its m.m.f. waves. There are three m.m.f. waves to be considered: that due to the field winding, FF, which is separately excited with direct current; that due to the 3-phase armature winding, FA; and their resultant, FR.

The machine considered will be a cylindrical-rotor machine with the 3-phase winding on the stator and the d.c.-excited field winding on the rotor. The generating mode of action will first be considered.
2.3.1 Generating mode of operation

- Open circuit operation
In this type of operation, the field m.m.f., FF is also the resultant m.m.f., FR, since on open-circuit there is no armature current and consequently FA is zero at all times and at all points in the air-gap.
The field m.m.f. is stationary with respect to the rotor winding, which is excited with direct current but moves, with the rotor, at synchronous speed past the stator winding. The diagram in Figure 2.3.11 represents this mode of operation.

![Figure 2.3.11: Open circuit operation](image)

- Unity power factor operation
Fig. 2.3.12 represents the operational mode when the armature winding is supplying current at unity power factor. Since the e.m.f. is caused by the resultant m.m.f., FR, this will have the same magnitude and position as previously in the open circuit operation.

![Figure 2.3.12 Unity power factor operation](image)

However, since in this case armature current flows, there will be an armature m.m.f., FA. The resultant m.m.f., FR, is the sum of FA and FF. Thus in this case FF and FR are different. The armature m.m.f. lags behind the resultant m.m.f. wave by $\pi/2$ radians. The armature m.m.f. moves at synchronous speed, so that the m.m.f.s FA and FF and their resultant FR all move at the same speed and in the same direction under steady conditions. At any time and at any point in the air-gap,

$$FR = FA + FF$$

Therefore

$$FA = FR - FF$$

To maintain the e.m.f. constant, the separate excitation has had to be increased in value. The effect of armature m.m.f. is therefore the same as that of an internal voltage drop.
The axis of the field m.m.f. is displaced by an angle $\sigma'$ in the direction of rotation, and as a result a torque is exerted on the rotor in the direction opposite to that of rotation. The rotor must be driven by a prime mover against this torque, so that machine absorbs mechanical energy and is therefore able to deliver electrical energy.
- **Power factor other than unity operation**

In the case of figure 2.3.13, the armature m.m.f. wave is displaced by an angle $\phi$ in the direction opposite to that of rotation as compared with its unity-Power-factor position. This change in the relative position of the armature m.m.f. wave brings it more into opposition with the field m.m.f., so that the latter must be further increased to maintain the resultant m.m.f. and e.m.f. constant.

![Figure 2.3.13 Lagging power factor operation](image)

Figure 2.3.13 Lagging power factor operation

Figure 2.3.14 illustrates the case when the phase currents are leading their respective e.m.f.s by a phase angle $\phi$. The change in the relative position of the armature m.m.f. wave gives it a component which aids the field m.m.f., which must then be reduced for a constant resultant m.m.f., FR, and e.m.f.

![Figure 2.3.14 Leading power factor operation](image)

Figure 2.3.14 Leading power factor operation

### 2.3.2 Motoring mode of operation

The power factor is again a way to classify the motoring operations. Figures 2.3.11-2.3.14 can be used to represent the motoring modes of operation as well.

Figure 2.3.11 shows the current and voltage waves for the synchronous motor on no-load. It is assumed that the no-load current is negligible and the armature m.m.f. zero. The field and resultant m.m.f.s are then identical as in the generator mode.

Figure 2.3.12 shows the diagram corresponding to motor operation at unity power factor. The armature m.m.f. is reversed at all points compared with the corresponding armature m.m.f. wave in the generator mode. The field m.m.f. is calculated by carrying out the subtraction FR - FA point by point round the air-gap. It will be noted that, compared with the no-load condition, the field m.m.f. is displaced by an angle $\sigma$ in the direction opposite to rotation, thus giving rise to a torque on the rotor acting in the direction of rotation. The machine thus delivers mechanical power, having absorbed electrical power from the supply.

Figs. 12.3(c) and (d) show the m.m.f. for operation at lagging and leading power factors respectively. Synchronous motors are operated from a constant
voltage supply, so that variation in the field excitation cannot affect the machine e.m.f. It follows that the power factor of the armature current must alter with variation of field excitation (this effect also occurs in generators connected in large constant-voltage systems). The input power factor becomes leading when the excitation is increased above the unity power factor condition, Similarly when the excitation is reduced the power factor becomes lagging.

2.4 Two Axis Representation
2.4.1 Direct and quadrature axes
If a machine has saliency, i.e. the rotor has salient poles, then the main field axis and the cross-field axis have different reluctances. The armature reaction m.m.f., Fa, can be resolved into two components Fd and Fq, as shown in Figure 2.41(b) for the m.m.f./flux/voltage transformation will now be different on these two component axes, known as the direct and quadrature axes, as less flux and induced e.m.f. will be produced on the q axis. This difference is reflected in the voltage phasor diagram of Figure 2.41(d) as two different reactances Xad and Xaq. The component of stator current acting on the d axis Id being associated with Xad and Iq with Xaq.

Figure 2.41: (a) Machine d and q axes, (b) The m.m.f. vectors resolved in d and q axes. Fd and Fq are components of Fa, (c) Current phasors resolved into Id and Iq components, (d) Voltages phasor diagram, Ef of figure 2.2 becomes Eq.

Because I has been resolved into two components it is no longer possible for a salient machine to be represented by an equivalent circuit. However, equations can be obtained for the Vd and Vq components of the terminal voltage Vt.
Eq can also be expressed in complex form as:

\[
Eq = V + IRL + jXL + j(IdXad + IqXaq) = V + IRL + j(IdXd + IqXq)
\]

Where:
\[Xd = Xad + XL\]
\[Xq = Xaq + XL\]

The transient behaviour of synchronous machines is often expressed in two-axis terms. The reason is due to the fact that to produce accurate voltage and current waveforms under transient conditions the two-axis method of representation with individually coupled coils on the d and q axes is necessary.

2.4.2 Output Power

The output power is given by \(VI\cos\phi\) where \(V\) is shown as \(Vt\) in Figure 2.41(d). If we redraw this figure neglecting \(RL\) and splitting \(IXL\) into two components \(IqXL\) and \(IdXL\) we obtain Figure 2.42. From Figure 2.42(b) it can be seen that

\[
E = V + jIdXd + jIqXq = V + jIXq + jId(Xd - Xq)
\]

(2.421)

Figure 2.42:(a) Current phasors and components, (b) Voltage phasors and components

As \(jld(Xd - Xq)\) is in phase with \(E\), and we know \(V\), \(I\) and \(\phi\) from the loading condition, \(IqXL\) can be drawn at right angles to the current \(I\) so that the direction of \(E\) can be determined. From this, the \(d\) and \(q\) axis directions can be drawn on both the current and voltage phasor diagrams and hence the remaining phasors can be established.

From Figure 2.42(a) the following equation can be deduced

\[
I \cos \phi = oa + ab = Id \sin \delta + Iq \cos \delta
\]
But

\[ V \sin \delta = I_q X_q \Rightarrow I_q = \frac{V \sin \delta}{X_q} \]

\[ V \cos \delta = E - I_d X_d \Rightarrow I_d = \frac{E - V \cos \delta}{X_d} \]

Hence

\[ P_{out} = V(1 \cos \varphi) = V(I_d \sin \delta + I_q \cos \delta) \]

\[ = V \left[ \frac{E - V \cos \delta}{X_d} \sin \delta + \frac{V \sin \delta}{X_q} \cos \delta \right] \]

\[ = \frac{EV - V^2 \cos \delta}{X_d} \sin \delta + \frac{V^2 \sin \delta}{X_q} \cos \delta \]

\[ = \frac{V^2 \sin \delta}{X_q} \cos \delta - \frac{V^2 \cos \delta}{X_d} \sin \delta + \frac{EV}{X_d} \sin \delta \]

\[ = V^2 \left( \frac{X_d \sin \delta \cos \delta - X_q \sin \delta \cos \delta}{X_d X_q} \right) + \frac{VE \sin \delta}{X_d} \]

\[ = V^2 \left( \frac{X_d \sin 2\delta - X_q \sin 2\delta}{2 X_d X_q} \right) + \frac{VE \sin \delta}{X_d} \]

\[ = \frac{VE}{X_d} \sin \delta + V^2 \left( \frac{X_d - X_q}{2 X_d X_q} \right) \sin 2\delta \] (2.422)

and

\[ \frac{dP}{d\delta} = \frac{VE}{X_d} \cos \delta + V^2 \left( \frac{X_d - X_q}{X_d X_q} \right) \cos 2\delta \] (2.423)

\[ = \text{synchronizing power coefficient} \]

The steady-State stability limit occurs at a \( \delta \) of less than 90° when \( X_d \) and \( X_q \) are very different. In steady-state calculations the effects of saliency may often be neglected.
2.5 Travelling wave Equations and phasor diagrams

2.5.1 MMF travelling wave equations

A synchronous machine wave, for both the generating and motoring modes of operation, at various power factors travel towards the $\theta$ direction at synchronous speed. Therefore, a traveling-wave equation, or function may represent each of its m.m.f.s. For example, the equation of the cosinusoidally distributed m.m.f. shown in Figure 2.51, which is traveling towards the $\theta$ direction at $\omega$ electrical radians per second is

$$F = F_m \cos(\omega t - \theta)$$  

(2.511)

Figure 2.51: Traveling wave M.M.F.

Considering generator action at a lagging power factor, the resultant m.m.f. is

$$FR = FR_m \cos(\omega t - \theta)$$  

(2.512)

The field m.m.f. is

$$FF = FF_m \cos(\omega t - \theta + \sigma')$$  

(2.513)

and the armature m.m.f. is

$$FA = FA_m \cos(\omega t - \theta - \frac{\pi}{2} - \phi')$$  

(2.514)

When the generator operates at a leading power factor, eqns. (2.512) and (2.513) still apply for the resultant and separate field m.m.f.s. The equation for the armature m.m.f. becomes

$$FA = FA_m \cos(\omega t - \theta + \frac{\pi}{2} + \phi')$$  

(2.515)

Considering now the motoring mode at a lagging power factor, the resultant m.m.f. is the same as for generator action, i.e. The separate field m.m.f. is the same as for generation action, i.e.

$$FR = FR_m \cos(\omega t - \theta)$$  

(2.516)

The separate field m.m.f. is

$$FF = FF_m \cos(\omega t - \theta - \sigma')$$  

(2.517)

and the armature m.m.f. is

$$FA = FA_m \cos(\omega t - \theta + \frac{\pi}{2} - \phi')$$  

(2.518)

When the power factor is leading, the armature m.m.f. becomes

$$FA = FA_m \cos(\omega t - \theta + \frac{\pi}{2} + \phi')$$  

(2.519)
2.5.2 MMF Phasor diagrams

The m.m.f.s of a synchronous machine may still be dealt with by means of phasor diagrams, since, under steady-state conditions, the relative positions of the waves do not alter. The m.m.f. phasor diagrams may be deduced directly from the traveling-wave equations of Section 2.51. For generator mode operation at a lagging power, at any particular point in the air-gap denoted by \( \theta = \theta_0 \) the traveling-wave equations of the m.m.f.s are

\[
FR' = FRm \times \cos(\omega t - \theta_0) \\
FF' = FFm \times \cos(\omega t - \theta_0 + \sigma^\prime) \\
FA' = FAm \times \cos(\omega t - \theta_0 - \frac{\pi}{2} - \phi^\prime)
\]

The above equations represent quantities varying sinusoidally with time. The corresponding complexor diagram is shown in Fig. 2.5.2(a), where the m.m.f.s \( FR', FF' \) and \( FA' \) are represented by the complexors \( FR, FF \) and \( FA \).

![Figure 2.5.2: M.M.F phasor diagrams for the Synchronous machine](image)

(a) Generator operation at a lagging power factor
(b) Generator operation at a leading power factor
(c) Motor operation at a lagging power factor
(d) Motor operation at a leading power factor
2.5.3 EMF phasor diagram
Regarding the diagrams shown in Figure 2.5.2, the principle of superposition may be applied. For a particular phase winding let "
EF = E.M.F. due to field m.m.f., FF
EA = E.M.F. due to armature m.m.f. FA
ER = Resultant e.m.f. due to the resultant m.m.f. FR

The relative phase angles of E EA and ER will be the same as those of FF, FA and FR. The m.m.f. and e.m.f. complexor diagrams are shown in Figs. 2.53(a) and (b) respectively for the case of a generator working at a lagging power factor. The resultant e.m.f., ER is shown as the complexor sum of EF and EA:

\[ ER = EF + EA \quad (2.531) \]

For a fixed value of separate excitation, E, will be constant. Any change of either the armature current or the load power factor would alter the resultant e.m.f. ER.

EF is customarily regarded as the e.m.f. since it does not alter with load and is also the terminal voltage on open-circuit.

The effect of the armature m.m.f. is treated, as an internal voltage drop. Since \( \frac{EA}{IA} \) is a constant, and since the phase of EA lags IA by 90°, this voltage may be represented as an inductive voltage drop, and the quotient as an inductive reactance, i.e.

\[ XA = \frac{EA}{IA} \quad (2.532) \]

Substituting EA in equation (2.531)

\[ EF = ER + IA \times XA \quad (2.533) \]
2.6 Voltage regulation
The voltage regulation of an alternator is normally defined as the rise in terminal voltage when a given load is thrown off. Thus, if \( E_f \) is the induced voltage on open-circuit and \( V \) is the terminal voltage at a given load, the voltage regulation is given by

\[
\text{Per - unit _ regulation} = \frac{E_f - V}{V}
\]

(2.70)

2.7 Equivalent Circuit under balanced short circuit conditions
Immediately after the application of a short circuit the main flux cannot change to a new value instantly, as it is linked with low-resistance circuits. As the flux remains unchanged initially, the stator currents are large and can flow only because of the creation of opposing currents in the rotor and damper windings by what is essentially transformer action. Owing to the higher resistance, the current induced in the damper winding decays rapidly and the armature current commences to fall. After this, the currents in the rotor winding and body decay, the armature reaction m.m.f. is gradually established, and the generated e.m.f. and stator current fall until the steady-state condition on short circuit is reached.

To represent the initial short-circuit conditions two new models must be introduced. Reactances are needed to represent the machine, the very initial conditions requiring what is called the subtransient reactance \( (X'' \) ) and the subsequent period the transient reactance \( (X' \) ).

In the following definitions it is assumed that the generator is on no-load prior to the application of the short-circuit and is of the round-rotor type.

Let the no-load phase voltage of the generator be \( E \) volts (rms)

Then, the subtransient reactance is

\[
(X'') = \frac{E}{O_b \sqrt{2}}
\]

where \( O_b / \sqrt{2} \) is the r.m.s. value of the subtransient current \( (I'' \) ).

The transient reactance is

\[
(X') = \frac{E}{O_a \sqrt{2}}
\]

where \( O_a / \sqrt{2} \) is the r.m.s. value of the transient current \( (I' \) ), and finally

\[
\frac{E}{O_c \sqrt{2}} = X_s
\]

Typical values of \( X'' \), \( I' \), and \( X_s \) for various types and sizes of machines are given in Table 2.7.
Table 2.7: Synchronous machine constants (at 60 Hz, per unit on rating)

<table>
<thead>
<tr>
<th>Type of Machine</th>
<th>X\textsubscript{s} or X\textsubscript{d}</th>
<th>X\textsubscript{q}</th>
<th>X\textasciitilde{}</th>
<th>X\textasciitilde{\textasciitilde{}}</th>
<th>X\textsubscript{2}</th>
<th>X\textsubscript{o}</th>
<th>RL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbo-alternator</td>
<td>1.2-2.0</td>
<td>1-1.5</td>
<td>0.2-0.35</td>
<td>0.17-0.25</td>
<td>0.17-0.25</td>
<td>0.04-0.14</td>
<td>0.003-0.008</td>
</tr>
<tr>
<td>Salient Pole (hydroelectric)</td>
<td>0.16-1.45</td>
<td>0.4-1.0</td>
<td>0.2-0.5</td>
<td>0.13-0.35</td>
<td>0.13-0.35</td>
<td>0.02-0.2</td>
<td>0.003-0.0015</td>
</tr>
<tr>
<td>Synchronous compensator</td>
<td>1.5-2.2</td>
<td>0.95-1.4</td>
<td>0.3-0.6</td>
<td>0.18-0.38</td>
<td>0.17-0.37</td>
<td>0.03-0.15</td>
<td>0.004-0.01</td>
</tr>
</tbody>
</table>

X\textsubscript{2} = Negative sequence reactance.
Y\textsubscript{0} = Zero sequence reactance.
X\textasciitilde{} and X\textasciitilde{\textasciitilde{}} are the direct axis quantities.
RL = a.c. resistance of the stator winding per phase.
If the machine is previously on load, the voltage applied to the equivalent reactance, previously E, is now modified due to the initial load volt-drop. Initially, the load current is I\textsubscript{L} and the terminal voltage is V. The voltage behind the transient reactance X\textasciitilde{} is

\[ E' = IL(ZL + jX\textasciitilde{}) \]
\[ = V + jIL \times X\textasciitilde{} \]

and hence the transient current on short circuit = E'\textasciitilde{jX\textasciitilde{}}
2.8 Synchronous generators parallel operation

Consider two machines A and B (as shown in Figure 2.8(a)), the voltages of which have been adjusted to equal values by means of the field regulators, and their speeds are slightly different. In Figure 2.8(b) the phase voltages are ERA etc., and the speed of machine A is $\omega_A$ radians per second and of B, $\omega_B$ radians per second. If the voltage phasors of A are considered stationary, those of B rotate at a relative velocity $(\omega_B - \omega_A)$ and hence there are resultant voltages across the switch S of $(ERA-ERB)$, which reduce to zero during each relative revolution. If the switch is closed at an instant of zero voltage, the machines are connected (synchronized) without the flow of large currents due to the resultant voltages across the armatures. When the two machines are in synchronism they have a common terminal-voltage, speed and frequency.

Figure 2.8: (a) generators in parallel, (b) corresponding phasor diagrams. (c) Machine A in Phase advance on machine B, (d) Machine B in Phase advance on machine A.

Consider two machines operating in parallel with $EA = EB$ and on no external load. If A tries to gain speed the phasor diagram in Figure 3.9(c) is obtained and $I = ER/(ZA + ZB)$. The circulating current $I$ lags ER by an angle $\tan^{-1}(X/R)$ and, as in most machines $X \gg R$, this angle approaches 90°. This current is a generating current for A and a motoring current for B. Hence A is generating power and tending to slow down and B is receiving power from A and speeding up. Therefore, A and B remain at the same speed, 'in step', or in synchronism.

Figure 3.9(d) shows the state of affairs when B tries to gain speed on A. The quality of a machine to return to its original operating state after a momentary disturbance is measured by the synchronizing power and torque. Normally, more than two generators operate in parallel in a power system. If the remaining machines in parallel are of such capacity that the presence of the generator under study causes no difference to the voltage and frequency of the other, they are said to comprise an infinite busbar system.

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2.9 Synchronous machines connected to large supply systems

In Britain, electrical energy is supplied to consumers from approximately 100 generating stations. The generating stations do not operate as isolated units but are interconnected by the national grid. The output of any single machine is therefore small compared with the total interconnected capacity. A machine connected to such a system, where the capacity of any one machine is small compared with the total interconnected capacity, is often said to be connected to infinite bus bars. The outstanding electrical characteristics of such bus bars are that they are constant-voltage constant-frequency bus bars. When the machine is connected to the infinite bus bar the terminal voltage and frequency becomes fixed at the values maintained by the rest of the system. Unless the machine is overloaded or under-excited, no change in the mechanical power supply, load or excitation will alter the terminal voltage or frequency. If the machine is acting as a generator and the mechanical driving power is increased the power output from the machine to the bus bars must increase. More detail regarding the infinite bus operation is provided in section 2.10

2.10 Generator operation on infinite bus bar

Figure 2.10 shows the schematic diagram of a machine connected to an infinite busbar along with the corresponding phasor diagram. The angle \( \delta \) between the \( E \) and \( V \) phasors is known as the load angle and is dependent on the power input from the turbine shaft.

\[
\begin{align*}
E & \quad \text{Xs} \quad I \quad V \\
\end{align*}
\]

(a) (b)

Figures 2.10 Synchronous machine connected to an infinite busbar, and its phasor diagram.

When connected to an infinite-busbar system the load delivered by the machine is not directly dependent on the connected load. By changing the turbine output, and hence \( I \), the generator can be made to take on any load that the operator desires subject to economic and technical limits. From the phasor diagram in Figure 2.10(b), the power delivered to the infinite busbar

\( = V \cos \phi \) per phase, but

\[
\frac{E}{\sin(90 + \phi)} = \frac{IX_s}{\sin \delta}
\]

hence

\[
I \cos \phi = \frac{E}{X_s} \sin \delta
\]

\[ \Rightarrow P = \frac{VE}{X_s} \sin \delta \quad (2.101) \]
Equation (2.11) is shown plotted in Figure 2.11. The maximum power is obtained at $\delta = 90^\circ$. If it becomes larger than $90^\circ$ due to an attempt to obtain more than $P$, increase in $\delta$ results in less power output and the machine becomes unstable and loses synchronism. Loss of synchronism results in the interchange of current surges between the generator and network as the poles of the machine pull into synchronism and then out again; i.e. the generator 'Pole slips'.

![Figure 2.11: Power-angle curve of a synchronous machine.](image)

If the power output of the generator is increased by small increments with the no-load voltage kept constant, the limit of stability occurs at $\delta = 90^\circ$ and is known as the steady-state stability limit. There is another limit of stability due to a sudden large change in conditions, such as caused by a fault, and this is known as the transient stability limit.

It is possible for the rotor to oscillate beyond $90^\circ$ a number of times. If these oscillations diminish, the machine is stable.

The synchronizing power coefficient

$$\frac{\partial P}{\partial \delta} \quad \text{Watts per radian}$$

and the synchronizing torque coefficient

$$\frac{1}{\frac{\partial P}{\partial \delta}} = \frac{\omega_s}{\frac{\partial P}{\partial \delta}}$$
2.11 Synchronous generator performance chart construction.
The synchronous machine, for which a performance chart is constructed and shown in Figure 2.11, has the following data.
60 MW, 0.8 pf, 75 MVA, 11.8 kV, 3000 r.p.m, max exciter current of 500 A, Xs=2.94 Ω/phase

The procedure of chart construction is described below:
With centre 0, a number of semicircles are drawn of radii equal to various MVA loadings, the most important being the 75 MVA circle. Arcs with 0' as centre are drawn with various multiples of 00' (or V) as radii to give the loci for constant excitations. Lines may also be drawn from 0 corresponding to various power factors, but for clarity only 0.8 p.f. lagging is shown. Thus the machine is generating vars.
The operational limits are fixed as follows. The rated turbine output gives a 60 MW limit which is drawn as shown, i.e. line efg, which meets the 75 MVA line at g. The MVA arc governs the thermal loading of the machine, i.e. the stator temperature rise, so that the output is decided by the MVA rating. At point h, the rotor heating becomes more decisive and the maximum excitation current allowable decides the arc hj. In this case assumed to be 2.5 p.u. The remaining limit is that governed by loss of synchronism at leading power factors. The theoretical limit is the line perpendicular to 00' at 0' (i.e. δ = 90'), but, in practice a safety margin is introduced to allow a further increase in load of either 10 or 20 per cent before instability. In Figure 2.11 a 10 per cent margin is used and is represented by ecd. It is constructed in the following manner. Considering point 'a' on the theoretical limit on the E = I p.u. arc, the power 0'a is reduced by 10 per cent of the rated power (i.e. by 6 MW) to 0'b; the operating point must, however, still be on the same E arc and b is projected to c, which is a point on the new limiting curve. This is repeated for several excitations, finally giving the curve ecd. The complete operating limit is shown in thick line and the operator should normally work within the area bounded by this line, provided the generator is running at rated voltage.
2.12 Power / angle characteristic of synchronous machine

Figure 2.12 is part of the general load diagram for a synchronous machine and shows the complexor diagram corresponding to generation into infinite busbar at a lagging power factor.

Figure 2.12: Generation diagram of a synchronous machine at lagging power factor

The power transfer is

\[ P = 3V \times I \times \cos \phi = 3V \times Ia \quad (2.121) \]

where V is the phase voltage and I is the phase current.

The promotion of the complexors of Figure 2.12 on the steady-state limit of stability line OD gives

\[ Ia \times Zs = Ef \times \cos(\psi - \sigma) - V \times \cos \psi \quad (2.122) \]

Substituting the expression for \( Ia \) obtained from equation (2.122) in equation (2.121) gives

\[ P = \frac{3V}{Zs} (Ef \times \cos(\psi - \sigma) - V \times \cos \psi) \quad (2.123) \]

Following the same procedure for motor action and he power transfer is found to be

\[ P = \frac{3V}{Zs} (V \times \cos \psi - Ef \times \cos(\psi - \sigma)) \quad (2.124) \]

Evidently eqn. (2.124) will cover both generator action and motor action if the power transfer \( P \) and the load angle \( \sigma \) are taken to be positive for generator action and negative for motor action.

Since, for steady-state operation, the speed of a synchronous machine is constant, the torque developed is

\[ T = \frac{P}{2 \pi \omega} = \frac{3V}{2 \pi \omega \times Zs} (Ef \times \cos(\psi - \sigma) - V \times \cos \psi) \quad (2.125) \]

In many synchronous machines \( Xs \approx R \), in which case.

\[ Zs \angle \psi = Xs \angle 90^\circ \]

When this approximation is permissible equation (2.123) becomes

\[ P = \frac{3V}{Zs} (Ef \times \cos(90^\circ - \sigma) - V \times \cos 90^\circ) \]

\[ = \frac{3V \times Ef}{Xs} \sin \sigma \]
Similarly equation (2.125) becomes

\[ T = \frac{3V \times Ef}{2\pi \omega \times Xs} \sin \sigma \]  

(2.127)

Usually stable operation can not be obtained beyond the 90° point, so that if the load angle exceeds it, the operation is dynamic with the machine either accelerating or decelerating.

### 2.13 Synchronising Power and Torque coefficients

A synchronous machine, whether a generator or a motor, when synchronized to infinite bus bar, has an inherent tendency to remain Synchronized. Consider a generator operation at a lagging power factor. At a steady load angle \( \delta \) the steady power transfer is \( P_0 \). Suppose that due to a transient disturbance, the rotor of the machine accelerates, so that the load angle increases by \( \delta \sigma \). This alters the operating point of the machine to a new constant-power line and the load on the machine increases to \( P_0 + \delta P \). Since the steady power input remains unchanged, this additional load retards the machine and brings it back to synchronism. Similarly, if owing to a transient disturbance, the rotor decelerates so that the load angle decreases, the load on the machine is thereby reduced to \( P_0 - \delta P \). This reduction in load causes the rotor to accelerate and the machine is again brought back to synchronism. Clearly the effectiveness of this inherent correcting action depends on the extent of the change in power transfer for a given change in load angle. A measure of this effectiveness is given by the synchronizing power coefficient, which is denoted as

\[ Ps = \frac{\partial P}{\partial \sigma} \]  

(2.140)

From equation (2.123),

\[ P = \frac{3V}{Zs} \left[ Ef \cos(\psi - \sigma) - V \cos \psi \right] \]  

(2.123)

So that

\[ Ps = \frac{\partial P}{\partial \sigma} = \frac{3V \times Ef}{Zs} \times \sin(\psi - \sigma) \]  

(2.141)

Similarly the synchronizing torque coefficient is defined as

\[ Ts = \frac{\partial T}{\partial \sigma} = \frac{1}{2\pi \times \omega \times no} \times \frac{\partial P}{\partial \sigma} \]  

(2.142)

Therefore,

\[ Ts = \frac{3}{2\pi \times \omega \times no} \times \frac{V \times Ef}{Zs} \times \sin(\psi - \sigma) \]  

(2.143)

In many synchronous machines \( Xs > R \), in which case equations (2.141) and (2.143) become

\[ Ps = \frac{3V \times Ef}{Xs} \cos \sigma \]  

(2.144)
Equations (2.144) and (2.145) show that the restoring action is greatest when \( \sigma = 0 \), i.e. on no-load. The restoring action is zero when \( \sigma = \pm 90^\circ \). At these values of load angle the machine would be at the steady state limit of stability and in a condition of unstable equilibrium. It is impossible, therefore, to run a machine at the steady-state limit of stability since its ability to resist small changes is zero, unless the machine is provided with a special fast-acting excitation system.

### 2.14 Oscillation of Synchronous machines

Normally the inherent stability of alternators when running in parallel quickly restores the steady-state condition, but if the effect is sufficiently great, the machine’s rotor may be subject to continuous oscillation and eventually break from synchronism.

This continuous oscillation of the rotor (periods of acceleration and deceleration) is sometimes known as phase swinging or hunting. Figure 2.14 shows the torque/load-angle characteristic of a synchronous generator. The steady input torque is \( T_o \), corresponding to a steady-state load angle \( \sigma_o \).

![Figure 2.14: Oscillation of synchronous machine connected to infinite busbar](image)

Suppose a transient disturbance occurs such as to make the rotor depart from the steady state by \( \sigma_1 \). Let \( \sigma_1 \) be sufficiently small to assume that the synchronizing torque is constant. Thus the torque/load-angle characteristic is assumed to be linear over the range of \( \sigma_1 \) considered.

Let \( T_s = \) Synchronizing torque coefficient (N-m/mech. rad)

\( \sigma_1 \) : Load angle deviation from steady-state position (mech. rad)

\( J \) = Moment of inertia of rotating system (kg-m^2)

Assuming that there is no damping.

\[
J \frac{d^2 \sigma_1}{dt^2} = -T_s \sigma_1 \tag{2.141}
\]

The solution of this differential equation is

\[
\sigma_1 = \sigma_0 \times \sin \left( \frac{T_s}{J} t + \psi \right) \tag{2.142}
\]
From eqn. (2.142), the frequency of undamped oscillation is

\[ f = \frac{1}{2\pi} \sqrt{\frac{T_s}{J}} \]  

(2.143)

Synchronous machines intended for operation on infinite busbars are provided with damping windings in order to prevent the sustained oscillations predicted by eqn. (2.143). The currents induced in the damping bars give a damping torque, which prevents continuous oscillation of the rotor.

### 2.15 Synchronous motors advantages and disadvantages

A synchronous motor will not develop a diving torque unless it is running at synchronous speed, since at any other speed the field poles will alternatively be acting on the effective N and S poles of the rotating field and only a pulsating torque will be produced. For starting, either (a) the induction motor principle or (b) a separate starting motor, must be used. If the latter method is used, the machine must be run up to synchronous speed and synchronized as an alternator. To obviate this trouble, synchronous motors are usually started as induction motors, and have a squirrel cage winding embedded in the rotor pole faces to give the required, action. When the machine has run up to almost synchronous speed the d.c. excitation is switched on to the rotor, and it then pulls into synchronism. The induction motor action then ceases. The starting difficulties of a synchronous motor severely limit its usefulness. It may only be used where the load may be reduced for starting and where starting is infrequent. Once started, the motor has the advantage of running at a constant speed with any desired power factor. Typical applications of synchronous motors are the driving of ventilation or pumping machinery where the machines run almost continuously. Synchronous motors are often run with no load to utilize their leading power factor characteristic for power factor correction or voltage control. In these applications the machine is called a synchronous phase modifier.

In general he principal advantages of the synchronous motor are:

1. The ease with which the power factor can be controlled. An over-excited synchronous motor having a leading power factor can be operated in parallel with induction motors having a lagging power factor, therefore improving the power factor of the supply system. Synchronous motors are sometimes run on no load for power-factor correction or for improving the voltage regulation of a transmission line. in such applications, the machine is referred to as a synchronous capacitor.

2. The speed is constant and independent of the load. This characteristic is mainly of use when the motor is required to drive another generator to generate a supply at a different frequency, as in frequency-changers.

The principal disadvantages are:

1. The cost per kilowatt is generally higher than that of an induction motor.

2. A d.c. supply is necessary for the rotor excitation. This is usually provided by a small d.c. shunt generator carried on an extension of the shaft.

3. Some arrangement must be provided for starting and synchronizing the motor.